RELIABILITY MODELING OF A BUTTER CHURNER AND CONTINUOUS BUTTER MAKING PRODUCTION SYSTEM

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Abstract

In the dairy plant, an investigation into the machine that makes butter was subjected to a reliability study in relation to the seasonal demand. In the process of expanding the butter churner into a machine that can make butter continuously, a more reliable operational model was devised. Both the models and the data acquired with MATLAB have been subjected to availability and reliability testing and analysis. In addition, the graphical analysis was carried out with the help of Code Blocks and Excel. A comparison of the two models was then covered as the final topic. It was discovered that (a) the extended model was superior to the current model, (b) the failure rate of the existing line increased, which implies that a new machine needs to be added to the line to share the load, which results in improved production, and (c) the failure rate of the extended model was lower than the failure rate of the existing model. (c) in order to maximise profits while simultaneously minimising losses The effectiveness of the system ought to be enhanced by performing routine maintenance during both the summer and the winter.

Keywords: Butter churner, continuous butter making, seasons, semi-Markov process, profit.

1. Introduction

As a result of high levels of "lifetime" engineering uncertainty, reliability engineering deals with predicting, preventing, and managing engineering failures. Costs of failures caused by equipment failure, parts costs, repairs, and personnel costs are all taken into account when reliability engineering is conducted. Industry engineers now put their effort on efficiency and high quality production. This can be achieved by improving system performance. When it comes to industrial applications on food production lines, ensuring a high level of reliability is highly important; however, reliability itself can be complex, many interconnected variables must be taken into account when guiding and assessing various levels of reliability.

Using maintenance regimes [9] processed site performance improvement in the dairy industry. [8] presented a case study on optimised performance of butter oil production. Based on real data [5] represented generation of wind power and electric power demand. Reliability analysis where operation is effected by temperature conditions was given by [2] and [1]. RAM analysis for modeling complex engineering systems was used by [6].

Introducing redundancy into a system can enhance its reliability. Redundancy with standby (redundant) units refers to the usage of additional units with the primary unit of the system, with the additional unit(s) becoming operational and performing all the desired functions with equivalent parameters upon the failure of the primary unit. Standby redundancy technique was used by several researchers to enchance system performance namely [3], [4], [7] etc. Work on standby units in a dairy industry was done by [10], [11] and [12].

Description of the systems

In model 1, the system which we have considered consists of a churner that works in both the seasons i.e., summer and winter. In winters, due to high demand system is always operating unless a failure occurs that can be due to electricity hault or any fault in the churner. In summers, due to less demand the system sometimes goes to cold standby state when there is no demand. In model 2, the system consists of churner and continuous butter making. Both the units starts to operate to accomodate the demand in winters, on the failure of any one unit the system works on reduced capacity. In summers, the butter churner is operative and CBM is in cold standby state, it operates on the failure of the churner. The system either goes to cold standby or maintenance state when there is no demand.

Methods

Both the models have been analyzed using semi-Markov process and regenerative point technique probabilistically.

2. Annotations

Table 1:

Notations of the model 1	
Notations	Descriptions
λ	Failure rate of the main unit i.e. Churner.
λ_1	Rate of electricity failure due to which churner stops operating.
γ	Rate at which churner goes to down state when demand is less than production.
δ	Rate when churner comes to operative state from a cold standby state.
α	Rate of going from winters to summers.
β	Rate of going from summers to winters.
ch	Main unit of the system i.e.ch.
S	Summer season.
W	Winter season.
Och	Main unit of the system is in operating state.
$d > p$	Demand is more than production.
$d < p$	Demand is less than production.
$CSch$	Main unit is in cold standby state.
$Frch$	Main unit is under repair.
$HCSch$	Main unit in cold standby state due to electricity hault.
$G(t), g(t)$	c.d.f. and p.d.f of time to repair of the main unit.
$G_1(t), g_1(t)$	c.d.f. and p.d.f of time to repair the electricity hault.
$G_2(t), g_2(t)$	c.d.f. and p.d.f of time to going back to operating state from down state.

3. Transition Probabilities and Mean Sojourn Time

Various states of the system are shown in figure 3.1 called as state transition diagram. Here, the states S_0, S_1, S_2 are operating states, S_5 is a cold standby state whereas, states S_3, S_4, S_6, S_7 are the failed states.

Transition Probabilites

- $dQ_{01}(t) = \beta e^{-(\alpha+\beta)(t)}dt$
- $dQ_{02}(t) = \alpha e^{-(\alpha+\beta)(t)}dt$
- $dQ_{13}(t) = \lambda_1 e^{-(\lambda+\lambda_1)(t)}dt$
- $dQ_{14}(t) = \lambda e^{-(\lambda+\lambda_1)(t)}dt$
- $dQ_{25}(t) = \gamma e^{-(\gamma+\lambda+\lambda_1)(t)}dt$
- $dQ_{26}(t) = \lambda_1 e^{-(\gamma+\lambda+\lambda_1)(t)}dt$

- $dQ_{27}(t) = \lambda e^{-(\gamma+\lambda+\lambda_1)(t)}dt$

The non-zero probabilities p_{ij} are as follows:

- p_{ij}=$Q_{ij}(\infty) = \int_0^\infty q_{ij}dt$
- $p_{01} = \frac{\beta}{\alpha+\beta}$
- p_{02}=$\frac{\alpha}{\alpha+\beta}$
- $p_{13} = \frac{\lambda_1}{\lambda+\lambda_1}$
- $p_{14} = \frac{\lambda}{\lambda+\lambda_1}$
- $p_{25} = \frac{\gamma}{\gamma+\lambda+\lambda_1}$
- $p_{26} = \frac{\lambda_1}{\gamma+\lambda+\lambda_1}$
- $p_{27} = \frac{\lambda}{\gamma+\lambda+\lambda_1}$
- $p_{31} = p_{62} = g *_1 (0)$
- $p_{41} = p_{72} = g * (0)$

From the above transition probabilities it is verified that:

- $p_{01} + p_{02} = 1$
- $p_{13} + p_{14} = 1$
- $p_{25} + p_{26} + p_{27} = 1$

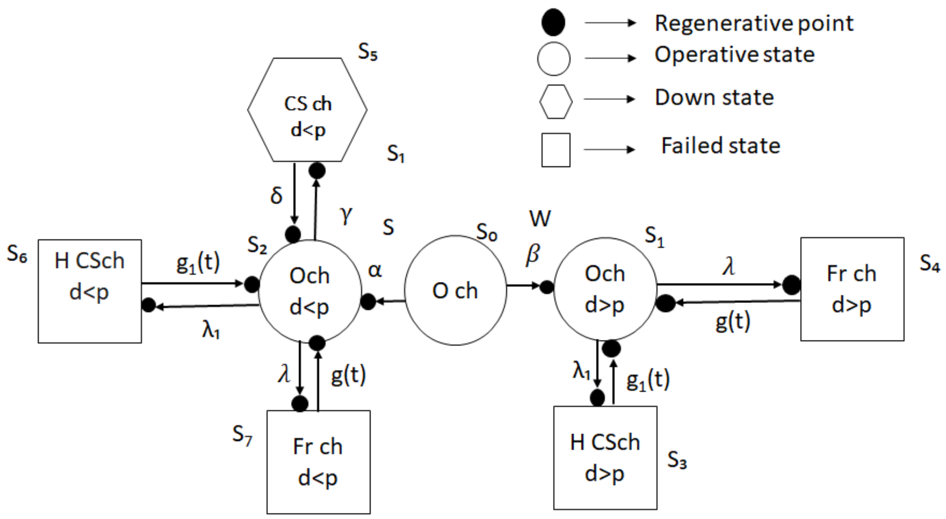

Figure 1: State Transition Diagram

The unconditional mean time taken by the system to transit for any regenerative state j when time is counted from the epoch of entrance into state i is mathematically state as:

- $m_{ij} = \int_0^\infty t dQ_{ij}(t)dt = -q_{ij}^*(0)$
- $m_{01} + m_{02} = \mu_0$
- $m_{13} + m_{14} = \mu_1$
- $m_{25} + m_{26} + m_{27} = \mu_2$

The mean sojourn time μ_i in the regenerative state $i$is defined as time of stay in that state before transition to any other state:

- $\mu_0 = \frac{1}{\alpha+\beta}$
- $\mu_1 = \frac{1}{\lambda+\lambda_1}$
- $\mu_2 = \frac{1}{\gamma+\lambda+\lambda_1}$
- $\mu_3 = \mu_6 = -g_1^*(0)$
- $\mu_4 = \mu_7 = -g^*(0)$
- $\mu_5 = \frac{1}{\delta}$

4. Mean Time to System Failure

The average duration between successive system failures, i.e. MTSF is defined as the expected time for which the system is in operation before it completely fails. Mean time to system failure

(MTSF) of the system is determined by considering failed state as absorbing state. When the system starts from the state 0, the mean time to system failure is:

$$T_0 = \lim_{s \longrightarrow 0} R^*(s) = \lim_{s \longrightarrow 0} \frac{1 - \phi_o^{**}(s)}{s} = \frac{N}{D}$$

where,
N=(μ_0+$\mu_1 p_{01}$)(1-p_{25})+(μ_2+$\mu_5 p_{25}$)(p_{02})
D=1-p_{25}

5. Availability Analysis of the System in Summers

Availability $A_i(t)$ is a measure that allows for a system to repair when failure occurs. The availability of the system is defined as the probability that the system is successful at time t. The long run availability of the system is given by

$$A_0^s = \lim_{s \longrightarrow 0}[sA_0^{*s}(s)] = \frac{N_1}{D_1}$$

where,
N_1=$\mu_2 p_{02}$
D_1=μ_2+$\mu_5 p_{25}$+$\mu_0 p_{26} + \mu_7 p_{27}$

6. Availability Analysis of the System in Winters

Availability $A_i(t)$ is a measure that allows for a system to repair when failure occurs. The availability of a system is defined as the probability that the system is successful at time t. The long run availability of the system is given by

$$A_0^w = \lim_{s \longrightarrow 0}[sA_0^{*w}(s)] = \frac{N_2}{D_2}$$

where,
N_2=$\mu_1 p_{01}$
D_2=μ_1+$\mu_4 p_{14}$+$\mu_3 p_{13}$

7. Busy Period Analysis for Repair in Summers

Busy period $B_i(t)$ in summers is defined as the probability that the repairman is busy at time t when the system entered to a regenerative state i. The total time in which the repairman is busy doing repair of the system in steady state is given by:

$$B_0^s = \lim_{s \longrightarrow 0}[sB_0^{*s}(s)] = \frac{N_3}{D_1}$$

where,
N_3=$p_{02}(p_{26}\mu_6 + p_{27}\mu_7)$
$D_1$is already defined above.

8. Busy Period Analysis for Repair in Winters

Busy period $B_i(t)$ in winters is defined as the probability that the repairman is busy at time t when the system entered to a regenerative state i. The total time in which the repairman is busy doing repair of the system in steady state is given by:

$$B_0^w = \lim_{s \longrightarrow 0}[sB_0^{*w}(s)] = \frac{N_4}{D_2}$$

where,
N_4=$p_{01}(W_3 p_{13} + W_4 p_{14})$
$D_2$is already defined above.

9. Expected Number of Repairs in Summers

Let $V_i(t)$ be the expected number of repairs in $(0,t)$ given that the system entered into regenerative state i at $i=0$. The expected number of repairs during summers in steady state is given by:

$$V_r = \lim_{s\longrightarrow 0} sV_r^{**}(s) = \frac{N_5}{D_1}$$

N_5=$p_{02}(1-p_{25})$
D_1 is already defined above in equation.

10. Expected Number of Repairs in Winters

Let $V_i(t)$ be the expected number of repairs in $(0,t)$ given that the system entered into regenerative state i at $i=0$. The expected number of repairs during summers in steady state is given by:

$$V_r = \lim_{s\longrightarrow 0} sV_r^{**}(s) = \frac{N_6}{D_2}$$

N_6=p_{01}
D_2 is already defined above in equation.

11. Profit Analysis of the System

Profit incurred to the system model in steady state is given by

$$P = (C_0A_0^s + C_1A_0^w) - (C_2B_0^s + C_3B_0^w + C_4V_0^s + C_5V_0^w)$$

where,
C_0=Revenue per unit up time in summers.
C_1=Revenue per unit up time in winters.
C_2=Cost per unit up time for which the repairman is busy for repair in summers.
C_3=Cost per unit up time for which the repairman is busy for repair in winters.
C_4=Cost per repair in summers.
C_5=Cost per repair in winters.

12. Graphical Analysis and Conclusion

For further numerical and graphical evaluation, let us assume the repair and failure rates to be exponentially distirbuted
$g(t) = \theta e^{-\theta(t)}, g_1(t) = \theta_1 e^{-\theta_1(t)}$

- $p_{01} = \frac{\beta}{\alpha+\beta}$
- p_{02}=$\frac{\alpha}{\alpha+\beta}$
- $p_{13} = \frac{\lambda_1}{\lambda+\lambda_1}$
- $p_{14} = \frac{\lambda}{\lambda+\lambda_1}$
- $p_{25} = \frac{\gamma}{\gamma+\lambda+\lambda_1}$
- $p_{26} = \frac{\lambda_1}{\gamma+\lambda+\lambda_1}$
- $p_{27} = \frac{\lambda}{\gamma+\lambda+\lambda_1}$
- $p_{31} = p_{62} = 1$
- $p_{41} = p_{72} = 1$
- $\mu_0 = \frac{1}{\alpha+\beta}$
- $\mu_1 = \frac{1}{\lambda+\lambda_1}$
- $\mu_2 = \frac{1}{\gamma+\lambda+\lambda_1}$
- $\mu_3 = \mu_6 = \frac{1}{\theta_1}$
- $\mu_4 = \mu_7 = \frac{1}{\theta}$
- $\mu_5 = \frac{1}{\delta}$

The parameters obtained using the original data collected from the Verka Milk Plant, Bathinda, Punjab.

Table 2:

Parameters obtained from data collected	
Parameters for model 1	Values
λ	.00045892
λ_1	.0002563
$g_1(t)$	.04213
$g(t)$	.062981
α	.0004314
β	.000526
δ	.000155
γ	.000955
C_0	830000
C_1	1030000
C_2	10500
C_3	12500
C_4	12000
C_5	15500

System effectiveness measures evaluated are given below:

Table 3:

Parameters obtained from data collected	
Parameters for model 1	Values
Mean time to system failure	9453.77 hrs
Availability in summers	.8975
Availability in winters	.8984
Busy period for repair in summers	.000485
Busy period for repair in winters	.0004204
Expected number of repairs in summers	.000217
Expected number of repairs in winters	.000031

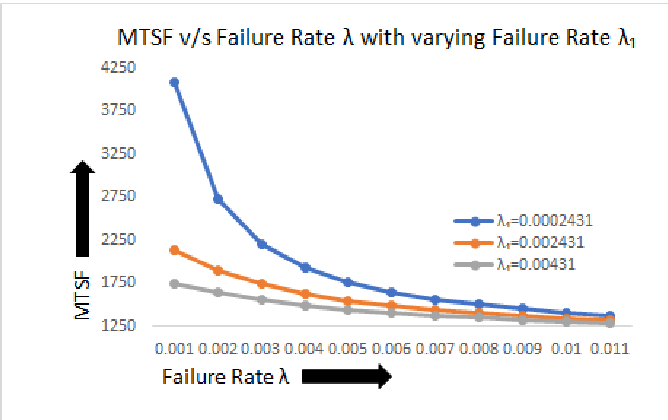

Figure 2: MTSF v/s Failure Rate

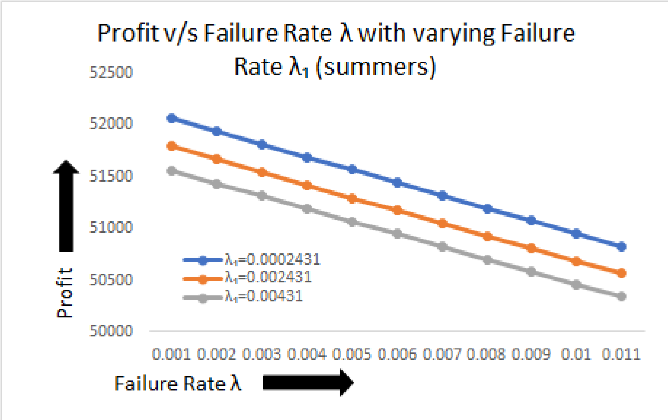

Figure 3: Profit v/s Failure Rate in Summers

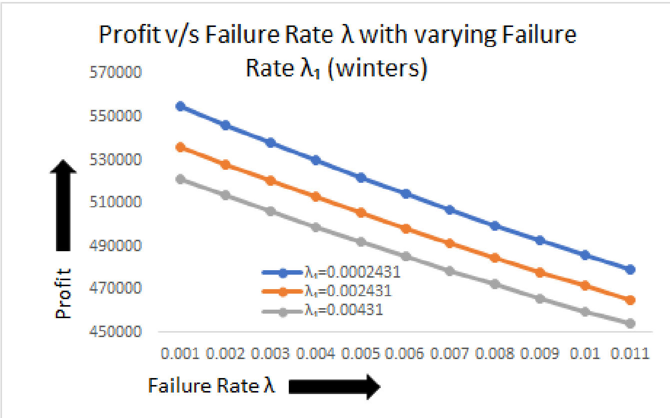

Figure 4: Profit v/s Failure Rate in Winters

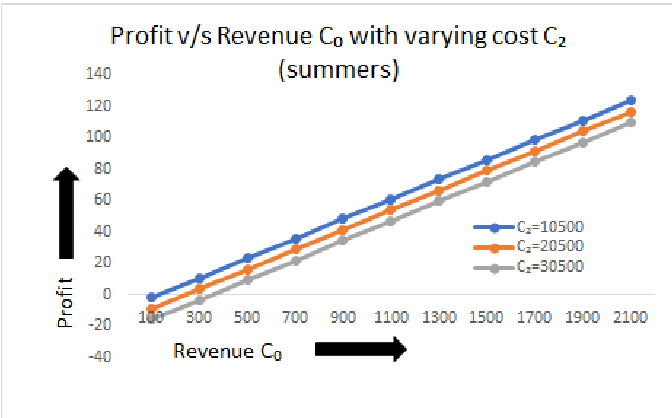

Figure 5: Profit v/s Failure Rate in Winters

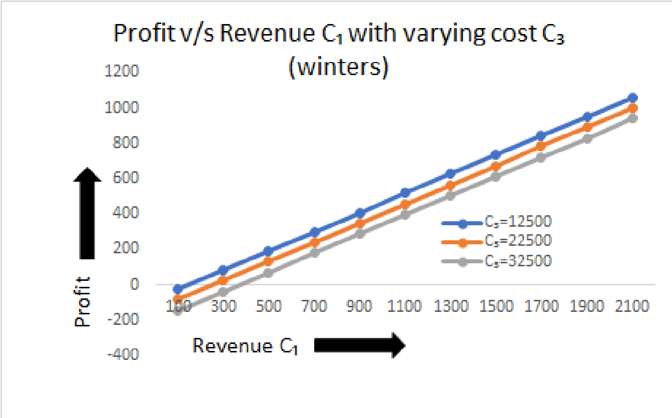

Figure 6: Profit v/s Failure Rate in Winters

Table 4:

	Notations of the model
Figures	Descriptions
5	Profit P1 increases as the revenue C_0 increases. C_2=10500; Profit >=< according to C_2, when C_2 is >=<Rs.275.53, similarly for C_2=20500 where cut off point is Rs.163.577 C_2=30500; where cut off point is Rs. 452.675
6	Profit P2 increases as the revenue C_1 increases. C_3=12500; Profit >=< according to C_3, when C_1 is >=<Rs.251.85, similarly for C_3=22500 where cut off point is Rs.140.469. C_3=32500; where cut off point is Rs. 429.089

Figure 3 and figure 4 depicts the trend of mean time to system failure and profit v/s the failure rate. It has been observed that as the failure rate λ of the system increases mean time to system failure and profit decreases. It also decreases on increasing failure rate λ_1. Figure 5,6 states that profit increases as the cost C_1 increases as well it increases with increasing profit C_3.

MODEL 2 Assumptions

Model 2 have the following assumptions:

- The system is operating at the initial stage.
- At the initial stage the churner is operating and continuous butter making is in a cold standby state.
- Both the systems operates during winters due to high demand.
- Only one unit is operating during summers due to less demand.
- In summers it also undergoes maintenance.
- The system sometimes goes to cold standby state in case of no demand in summers.
- The repair is done on the failure of the system.
- Repair rates are assumed to have arbitrary distribution.
- Failure rates are taken to be exponentially distributed.

- After repair the system operates as new.
- The system goes to failed state either on the failure of the churner or due to hault in the electricity.

13. Annotations for model 2

Table 5:

Notations of the model 2	
Notations	Descriptions
λ	Failure rate of the churner.
λ_1	Failure rate of the continuous butter making.
γ	Rate at which churner goes to down state when demand is less than production.
δ	Rate when churner comes to operative state from a cold standby state.
α	Rate of going to winters.
β	Rate of going to summers.
ch	Unit churner of the system.
cbm	Unit continuous butter making of the system.
S	Summer season.
W	Winter season.
Och	Churner is in operating state.
$Ocbm$	CBM is in operating state.
$d > p$	Demand is more than production.
$d < p$	Demand is less than production.
$CSch$	Main unit is in a cold standby state.
$CScbm$	CBM is in a cold standby state.
$Frch$	Churner is under repair.
$HCSch$	Churner is in cold standby state due to electricity hault.
$G(t), g(t)$	c.d.f. and p.d.f of time to repair of the churner.
$G_1(t), g_1(t)$	c.d.f. and p.d.f of time to repair of CBM.
$G_2(t), g_2(t)$	c.d.f. and p.d.f of time to going back to operating state from maintenance.

14. Model 2

15. Annotations for model 2

16. Transition Probabilites and Mean Sojourn Time

Various states of the system are shown in figure 1.5 called as state transition diagram. Here, the states S_0, S_1, S_2, S_3, S_5 are operating states, S_4 is a cold standby state whereas, states S_9, S_{10} are the reduced capacity states and rest are failed states.

- $dQ_{01}(t) = \beta e^{-(\alpha+\beta)(t)}dt$
- $dQ_{02}(t) = \alpha e^{-(\alpha+\beta)(t)}dt$
- $dQ_{19}(t) = \lambda_1 e^{-(\lambda+\lambda_1)(t)}dt$
- $dQ_{1,10}(t) = \lambda e^{-(\alpha+\beta)(t)}dt$
- $dQ_{23}(t) = \lambda_2 e^{(\lambda+\lambda_2+\gamma)}dt$
- $dQ_{24}(t) = \gamma e^{(\lambda+\lambda_2+\gamma)}dt$
- $dQ_{25}(t) = \lambda e^{(\lambda+\lambda_2+\gamma)}dt$
- $dQ_{32}(t) = g_2(t)e^{-\lambda(t)}dt$
- $dQ_{3,13}(t) = \lambda e^{-\lambda(t)}G\bar{(}t)dt$
- $dQ_{37}^{(13)}(t) = (\lambda e^{-\lambda(t)}(c)1)g_2(t)dt$

- $dQ_{42}(t) = \delta e^{-\delta(t)}dt$
- $dQ_{52}(t) = g(t)e^{-\lambda_1(t)}dt$
- $dQ_{56}(t) = \lambda_1 e^{-\lambda_1(t)}\bar{G(t)}dt$
- $dQ_{57}^{(6)}(t) = (\lambda_1 e^{-\lambda_1(t)})g(t)dt$
- $dQ_{67}(t) = g_2(t)dt$
- $dQ_{72}(t) = g_1(t)e^{-\lambda(t)}dt$
- $dQ_{78}(t) = \lambda e^{-\lambda(t)}\bar{G_1(t)}dt$
- $dQ_{75}^{(8)} = (\lambda e^{-\lambda(t)}(c)1)g_1(t)dt$
- $dQ_{91}(t) = g_1(t)e^{-\lambda(t)}dt$
- $dQ_{9,12}(t) = \lambda e^{-\lambda(t)}\bar{G_1(t)}dt$
- $dQ_{9,10}^{(12)}(t) = (\lambda e^{-\lambda(t)}(c)1)g_1(t)dt$
- $dQ_{10,1}(t) = g(t)e^{-\lambda_1(t)(t)}dt$
- $dQ_{10,11}(t) = \lambda_1 e^{-\lambda_1(t)}\bar{G(t)}dt$
- $dQ_{10,9}^{(11)}(t) = (\lambda_1 e^{-\lambda_1(t)}(c)1)g(t)dt$
- $dQ_{13,7}(t) = g_2(t)dt$
- $dQ_{12,10}(t) = g_1(t)dt$

The non-zero probabilities p_{ij} are as follows:

- p_{ij}=$Q_{ij}(\infty) = \int_0^\infty q_{ij}dt$
- $p_{01} = \frac{\beta}{\alpha+\beta}$
- $p_{02} = \frac{\alpha}{\alpha+\beta}$
- $p_{19} = \frac{\lambda_1}{\lambda+\lambda_1}$
- $p_{1,10} = \frac{\lambda}{\lambda+\lambda_1}$
- $p_{23} = \frac{\lambda_2}{\lambda+\lambda_2+\gamma}$
- $p_{24} = \frac{\gamma}{\lambda+\lambda_2+\gamma}$
- $p_{25} = \frac{\lambda}{\lambda+\lambda_2+\gamma}$
- $p_{32} = g_2^*(\lambda)$
- $p_{3,13} = p_{37}^{(13)} = 1 - g_2^*(\lambda)$
- $p_{52} = g_2^{(*)}(\lambda_1)$
- $p_{56} = p_{57}^{(6)} = 1 - g_2^{(*)}(\lambda_1)$
- $p_{72} = g_1^{(*)}(\lambda)$
- $p_{78} = p_{75}^{(8)} = 1 - g_1^{(*)}(\lambda)$
- $p_{91} = g_1^{(*)}(\lambda)$
- $p_{9,12} = p_{9,10}^{(12)} = 1 - g_1^{(*)}(\lambda)$
- $p_{10,1} = g^{(*)}(\lambda_1)$
- $p_{10,11} = p_{10,9}^{(11)} = 1 - g^{(*)}(\lambda_1)$

From the above transition probabilities it is verified that:

- $p_{01} + p_{02} = 1$
- $p_{19} + p_{1,10} = 1$
- $p_{23} + p_{24} + p_{25} = 1$
- $p_{32} + p_{3,13} = 1$
- $p_{32} + p_{37}^{(13)} = 1$
- $p_{52} + p_{56} = 1$
- $p_{52} + p_{57}^{(6)} = 1$
- $p_{72} + p_{78} = 1$
- $p_{72} + p_{75}^{(8)} = 1$
- $p_{91} + p_{9,12} = 1$
- $p_{91} + p_{9,10}^{(12)} = 1$
- $p_{10,1} + p_{10,11} = 1$
- $p_{10,1} + p_{10,9}^{(11)} = 1$

The unconditional mean time taken by the system to transit for any regenerative state j when it (time) is counted from the epoch of entrance into state $i$is mathematically state as:

- $m_{ij} = \int_0^\infty t dQ_{ij}(t)dt = -q_{ij}^*(0)$
- $m_{01} + m_{02} = \mu_0$
- $m_{19} + m_{1,10} = \mu_1$
- $m_{23} + m_{24} + m_{25} = \mu_2$
- $m_{32} + m_{3,13} = \mu_3$
- $m_{32} + m_{37}^{(13)} = K_2$
- $m_{52} + m_{56} = \mu_5$
- $m_{52} + m_{57}^{(6)} = K$
- $m_{72} + m_{75} = \mu_7$
- $m_{72} + m_{75}^{(8)} = K_1$
- $m_{91} + m_{9,12} = \mu_9$
- $m_{91} + m_{9,10}^{(12)} = K_1$
- $m_{10,1} + m_{10,11} = \mu_{10}$
- $m_{10,1} + m_{10,11}^{(11)} = K$

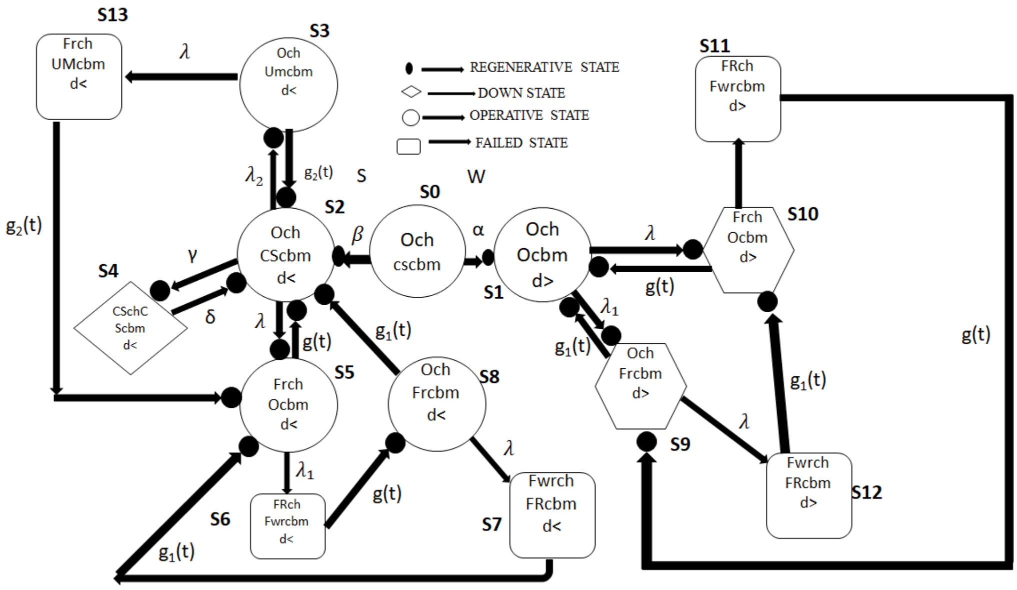

Figure 7: Model 2: State Transition Diagram

The mean sojourn time μ_i in the regenerative state $i$is defined as time of stay in that state before transition to any other state:

- $\mu_0 = \frac{1}{\alpha+\beta}$
- $\mu_1 = \frac{1}{\lambda+\lambda_1}$
- $\mu_2 = \frac{1}{\gamma+\lambda+\lambda_2}$
- $\mu_3 = \frac{1-g_2^*(\lambda_1)}{\lambda_1}$
- $\mu_4 = \frac{1}{\delta}$
- $\mu_5 = \frac{1-g^*(\lambda_1)}{\lambda}{}_1$
- $\mu_7 = \mu_9 = \frac{1-g_1^{(*)}(\lambda)}{\lambda}$
- $\mu_{10} = \frac{1-g^*(\lambda_1)}{\lambda_1}$
- $\mu_{11} = \int_0^\infty G(\bar{t})dt$
- $\mu_{12} = \int_0^\infty G_1(\bar{t})dt$

17. Mean Time to System Failure for Model 2

The average duration between successive system failures, i.e. MTSF is defined as the expected time for which the system is in operation before it completely fails. Mean time to system failure (MTSF) of the system is determined by considering failed state as absorbing state. When the system starts from the state 0, the mean time to system failure is:

$$T_0 = \lim_{s \longrightarrow 0} R^*(s) = \lim_{s \longrightarrow 0} \frac{1-\phi_o^{**}(s)}{s} = \frac{N}{D}$$

where,
D=$p_{19}p_{23}p_{32}p_{91} - p_{24} - p_{25}p_{52} - p_{19}p_{91} - p_{10,1}p_{1,10} - p_{23}p_{32} + p_{19}p_{24}p_{91} + p_{19}p_{25}p_{52}p_{91} + p_{23}p_{32}p_{10,1}p_{1,10} + p_{24}p_{10,1}p_{1,10} + p_{25}p_{52}p_{10,1}p_{1,10} + 1$
$N = \mu_0(p_{23}p_39 + p_{25}p_{56} - p_{19}p_{23}p_39p_{91} - p_{19}p_{25}p_{56}p_{91} - p_{23}p_39p_{10,1}p_{1,10} - p_{25}p_{56}p_{10,1}p_{1,10}) + \mu_1(p_{91} + p_{01}p_{9,12} - p_{23}p_{32}p_{91} - p_{24}p_42p_{91} - p_{25}p_{52}p_{91} - p_{02}p_{23}p_39p_{91} - p_{02}p_{25}p_{56}p_{91} - p_{01}p_{23}p_{32}p_{9,12} - p_{01}p_{24}p_42p_{9,12} - p_{01}p_{25}p_{52}p_{9,12}) + (\mu_2 + \mu_4p24)(p_42 - p_{19}p_42p_{91} - p_42p_{10,1}p_{1,10} - p_{01}p_{19}p_42p_{9,12} - p_{01}p_42p_{1,10}p_{10,11}) + \mu_3(p_{02}p_{23} - p_{02}p_{19}p_{23}p_{91} - p_{02}p_{23}p_{10,1}p_{1,10}) + \mu_5(p_{02}p_{25} - p_{02}p_{19}p_{25}p_{91} - p_{02}p_{25}p_{10,1}p_{1,10}) + \mu_9(p_{01}p_{19} - p_{01}p_{19}p_{23}p_{32} - p_{01}p_{19}p_{24}p_42 - p_{01}p_{19}p_{25}p_{52}) + \mu_{10}(p_{01}p_{1,10} - p_{01}p_{23}p_{32}p_{1,10} - p_{01}p_{24}p_42p_{1,10} - p_{01}p_{25}p_{52}p_{1,10})$

18. Reliability Measures

18.1. Availability Analysis in Summers

Availability $A_i(t)$ is a measure that allows for a system to repair when failure occurs. The availability of a system is defined as the probability that the system is successful at time t. The long run availability of the system is given by

$$A_0^s = \lim_{s \longrightarrow 0}[sA_0^{*s}(s)] = \frac{N_1}{D_1}$$

where,

$N_1 = \mu_0 + \mu_2 p_{02} + \mu_3 p_{02} p_{23} + \mu_5 p_{02} p_{25} - \mu_0 p_{23} p_{32} - \mu_0 p_{24} - \mu_0 p_{25} p_{52} - \mu_0 p_{57}^{(6)} p_{75}^{(8)} + \mu_7 p_{02} p_{23} p_{37}^{(13)} + \mu_7 p_{02} p_{25} p_{57}^{(6)} - \mu_0 p_{23} p_{37}^{(13)} p_{72} - \mu_2 p_{02} p_{57}^{(6)} p_{75}^{(8)} - \mu_0 p_{25} p_{57}^{(6)} p_{72} + \mu_5 p_{02} p_{23} p_{37}^{(13)} p_{75}^{(8)} - \mu_3 p_{02} p_{23} p_{57}^{(6)} p_{75}^{(8)} + \mu_0 p_{23} p_{32} p_{57}^{(6)} p_{75}^{(8)} - \mu_0 p_{23} p_{37}^{(13)} p_{52} p_{75}^{(8)} + \mu_0 p_{24} p_{57}^{(6)} p_{75}^{(8)}$

$D_1 = (\mu_2 + \mu_4 p_{24})(1 - p_{57}^{(6)} p_{75}^{(8)}) + \mu_3 (p_{23} p_{72} + p_{23} p_{52} p_{75}^{(8)}) + \mu_5 (p_{75}^{(8)} + p_{25} p_{72} - p_{23} p_{32} p_{75}^{(8)} - p_{24} p_{75}^{(8)}) + \mu_7 (p_{57}^{(6)} - p_{23} p_{32} p_{57}^{(6)} + p_{23} p_{37}^{(13)} p_{52} - p_{24} p_{57}^{(6)})$

18.2. Availability Analysis in Winters when the System Works at Full Capacity

The availability of a system is defined as the probability that the system is successful at time t. The long run availability of the system is given by

$$A_0^s = \lim_{s \longrightarrow 0}[sA_0^{*s}(s)] = \frac{N_2}{D_2}$$

where,

$N_2 = \mu_0 + \mu_1 p_{01} - \mu_0 p_{19} p_{91} - \mu_0 p_{10,1} p_{1,10} - \mu_0 p_{10,9}^{(11)} p_{9,10}^{(12)} - \mu_0 p_{91} p_{10,9}^{(11)} p_{1,10} - \mu_1 p_{01} p_{10,9}^{(11)} p_{9,10}^{(12)} - \mu_0 p_{19} p_{10,1} p_{9,10}^{(12)}$

$D_2 = \mu_1 (p_{10,1} + p_{91} p_{10,9}) + \mu_9 (p_{10,9} + p_{19} p_{10,1}) + \mu_{10} (p_{1,10} + p_{19} p_{9,10})$

18.3. Availability Analysis in Winters when the System Operates at Reduced Capacity

Availability of the system when it operates at reduced capapcity is given by

$$A_0^w = \lim_{s \longrightarrow 0}[sA_0^{*w}(s)] = \frac{N_3}{D_2}$$

where,

$N_3 = p_{01}(\mu_9 p_{19} + \mu_{10} p_{1,10} + \mu_9 p_{10,9}^{(11)} p_{1,10} + \mu_{10} p_{19} p_{9,10}^{(12)})$

D_2 is already defined above.

18.4. Busy Period Analysis for Repair in Summers

Busy period $B_i(t)$ in summers is defined as the probability that the repairman is busy at time t when the system entered to a regenerative state i. The total time in which the repairman is busy doing repair of the system in steady state is given by:

$$B_0^{sr} = \lim_{s \longrightarrow 0}[sB_0^{*sr}(s)] = \frac{N_4}{D_1}$$

where,

$N_4 = p_{02}(\mu_5 p_{25} + \mu_7 p_{23} p_{37}^{(13)} + \mu_7 p_{25} p_{57}^{(6)} + \mu_5 p_{23} p_{37}^{(13)} p_{75}^{(8)})$

$D_2$is already defined above.

18.5. Busy Period for Maintenance in Summers

Busy period $B_i(t)$ in summers for maintenance is obtained. The total time in which the repairman is busy doing repair of the system in steady state is given by:

$$B_0^{sm} = \lim_{s \longrightarrow 0}[sB_0^{*sm}(s)] = \frac{N_5}{D_2}$$

where,

$N_5 = -\mu_3 p_{02} p_{23} (p_{57}^{(6)} p_{75}^{(8)} - 1)$

$D_2$is already defined above.

18.6. Busy Period Analysis for Repair in Winters

Busy period for repair in winters is obtained as given below:
The total time in which the repairman is busy doing repair of the system in steady state is given by:

$$B_0^{wr} = \lim_{s\longrightarrow 0}[sB_0^{*wr}(s)] = \frac{N_6}{D_2}$$

where,
$N_6 = p_{01}(\mu_9 p_{19} + \mu_1 0 p_{1,10} + \mu_9 p_{10,9}^{(11)} p_{1,10} + \mu_{10} p_{19} p_{9,10}^{(12)})$
$D_2$is already defined above.

18.7. Expected Number of Repairs in Summers

Let $V_i(t)$ be the expected number of repairs in $(0,t)$ given that the system entered into regenerative state i at $i = 0$.
The expected number of repairs during summers in steady state is given by:

$$V^{sr} = \lim_{s\longrightarrow 0} sV^{sr}(s) = \frac{N_7}{D_1}$$

$N_7 = p_{02}(p_{25}p_{52} + p_{25}p_{57}^{(6)} + p_{23}p_{37}^{(13)}p_{72} + p_{23}p_{37}^{(13)}p_{75}^{(8)} + p_{25}p_{57}^{(6)}p_{72} + p_{25}p_{57}^{(6)}p_{75}^{(8)} + p_{23}p_{37}^{(13)}p_{52}p_{75}^{(8)} + p_{23}p_{37}^{(13)}p_{57}^{(6)}p_{75}^{(8)})$
D_1 is already defined above.

18.8. Expected Number of Maintenances in Summers

Let $V_i(t)$ be the expected number of maintenances. The expected number of repairs during summers in steady state is given by:

$$V^{sm} = \lim_{s\longrightarrow 0} sV^{sm}(s) = \frac{N_8}{D_1}$$

$N_8 = -(p_{32} + p_{37}^{(13)})p_{02}p_{23}(p_{57}^{(6)}p_{75}^{(8)} - 1)$
D_1 is already defined above.

18.9. Expected Number of Repairs in Winters

Let $V_i(t)$ be the expected number of repairs in winters. The expected number of repairs during summers in steady state is given by:

$$V^{wr} = \lim_{s\longrightarrow 0} sV^{wr}(s) = \frac{N_9}{D_2}$$

$N_9 = p_{01}(p_{19}p_{91} + p_{10,1}p_{1,10} + p_{10,9}p_{1,10} + p_{19}p_{9,10}^{(12)} + p_{91}p_{10,9}p_{1,10} + p_{19}p_{10,1}p_{9,10}^{(12)} + p_{19}p_{10,9}p_{9,10}^{(12)} + p_{10,9}p_{1,10}p_{9,10})$
D_2 is already defined above.

19. Profit Analysis of the System

Profit incurred to the system model in steady state is given by

$$P = (C_0 A_0^s + C_1 A_0^{wf} + C_2 A_0^{wr}) - (C_3 B_0^s + C_4 B_0^w + C_5 B_0^{sm} + C_6 V_0^{sr} + C_7 V_0^w + C_8 V_0^{sm})$$

where,
C_0=Revenue per unit up time in summers.
C_1=Revenue per unit up time in winters when the system operates at full capacity.
C_2=Revenue per unit up time in winters when the system operates at reduced capacity.
C_3=Cost per unit up time for which the repairman is busy for repair in summers.
C_4=Cost per unit up time for which the repairman is busy for repair in winters.
C_5=Cost per unit up time for which the repairman is busy for maintenance in summers.
C_6=Cost per repair in summers.
C_7=Cost per repair in winters.
C_8=Cost per maintenance in summers.

20. Graphical Analysis and Conclusion

For further numerical and graphical evaluation, let us assume the repair and failure rates to be exponentially distirbuted
$g(t) = \theta e^{-\theta(t)}, g_1(t) = \theta_1 e^{-\theta_1(t)}, g_2(t) = \theta_2 e^{-\theta_2(t)}$

- $p_{01} = \frac{\beta}{\alpha+\beta}$
- $p_{02} = \frac{\alpha}{\alpha+\beta}$
- $p_{19} = \frac{\lambda_1}{\lambda+\lambda_1}$
- $p_{1,10} = \frac{\lambda}{\lambda+\lambda_1}$
- $p_{23} = \frac{\lambda_2}{\lambda+\lambda_2+\gamma}$
- $p_{24} = \frac{\gamma}{\lambda+\lambda_2+\gamma}$
- $p_{25} = \frac{\lambda}{\lambda+\lambda_2+\gamma}$
- $p_{32} = \frac{\lambda_1}{\lambda_1+\theta_2}$
- $p_{37}^{(13)} = p_{3,13} = \frac{\theta_2}{\lambda_1+\theta_2}$
- $p_{52} = \frac{\lambda_1}{\lambda_1+\theta}$
- $p_{57}^{(6)} = p_{56} = \frac{\theta}{\lambda_1+\theta}$
- $p_{72} = \frac{\lambda}{\lambda+\theta_1}$
- $p_{75}^{(8)} = p_{78} = \frac{\theta_1}{\lambda+\theta_1}$
- $p_{91} = \frac{\lambda}{\lambda+\theta_1}$
- $p_{9,10}^{(12)} = p_{9,12} = \frac{\theta_1}{\lambda+\theta_1}$
- $p_{10,1} = \frac{\lambda_1}{\lambda_1+\theta}$
- $p_{10,11}^{(12)} = p_{10,12} = \frac{\theta}{\lambda_1+\theta}$
- $\mu_0 = \frac{1}{\alpha+\beta}$
- $mu_1 = \frac{1}{\lambda+\lambda_1}$
- $\mu_2 = \frac{1}{\gamma+\lambda+\lambda_2}$
- $\mu_3 = \frac{\theta_2}{\lambda_1(\lambda_1+\theta_2)}$
- $\mu_4 = \frac{1}{\delta}$
- $\mu_5 = \mu_{10} = \frac{\theta}{\lambda_1(\lambda_1+\theta)}$
- $\mu_7 = \mu_9 = \frac{\theta_1}{\lambda(\lambda+\theta_1)}$
- $\mu_{13} = \frac{1}{\theta_2}$
- $\mu_8 = \mu_{12} = \frac{1}{\theta_1}$
- $\mu_6 = \frac{1}{\theta}$

The parameters obtained using the original data collected from the Verka Milk Plant, Bathinda, Punjab.

Table 6:

Parameters obtained from data collected	
Parameters for model 1	Values
λ	.00045892
λ_1	.0004567
λ_2	0.000246572
$g_1(t)$	.06312
$g(t)$	.062981
$g_2(t)$	0.002628867
α	.000562
β	.0004314
δ	.000955
γ	.000155
C_0	830000
C_1	1030000
C_2	61660
C_3	10500
C_4	12500
C_5	15500

C_6	19500
C_7	6400
C_8	7000

System effectiveness measures evaluated are given below:

Table 7:

Parameters obtained from data collected	
Parameters for model 2	Values
Mean time to system failure	99682.28 hrs
Availability in summers	0.985
Availability in winters when system operates at full capacity	.989
Availability in winters when system operates at reduced capacity	.001435
Busy period for repair in summers	.003814
Busy period for maintenance in summers	.038744
Busy period for repair in winters	.007864
Expected number of repairs in summers	.000242
Expected number of maintenances in summers	.000120
Expected number of repairs in winters	.000499

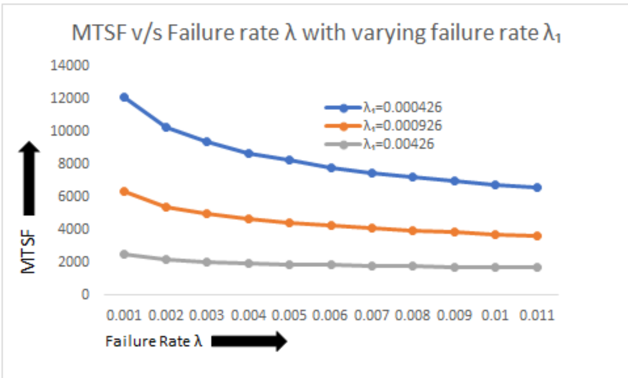

Figure 8: MTSF v/s Failure Rate

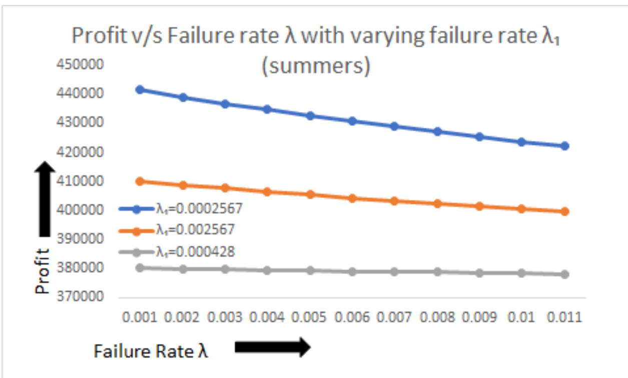

Figure 9: Profit v/s Failure Rate in Summers

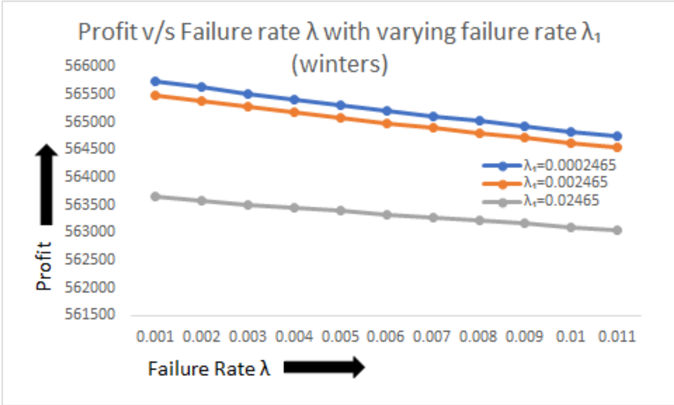

Figure 10: Profit v/s Failure Rate in Winters

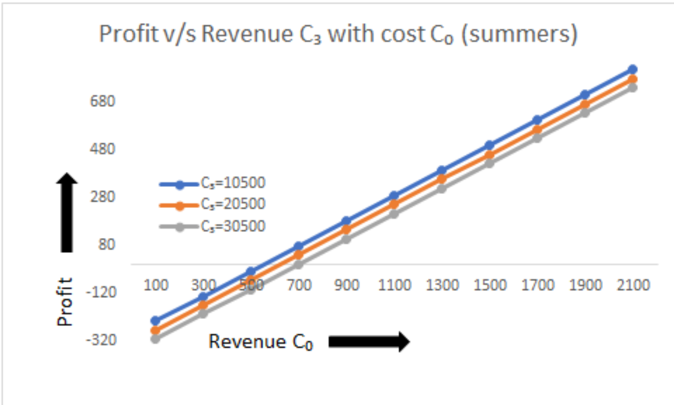

Figure 11: Profit v/s Failure Rate in Winters

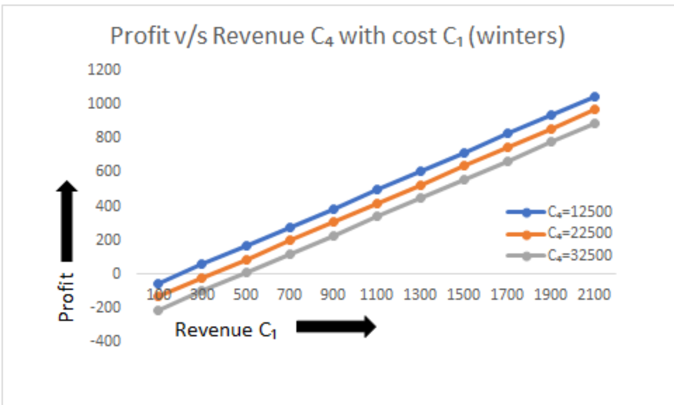

Figure 12: Profit v/s Failure Rate in Winters

Table 8:

Notations of the model	
Figures	Descriptions
11	Profit P1 increases as the revenue C_0 increases. C_3=10500; Profit >=< according to C_3, when C_3 is >=<Rs.645.34, similarly for C_3=20500 where cut off point is Rs.573.039 C_3=30500; where cut off point is Rs. 500.7389
12	Profit P2 increases as the revenue C_1 increases. C_4=12500; Profit >=< according to C_4, when C_1 is >=<Rs.203.345, similarly for C_4=22500 where cut off point is Rs.460.203. C_4=32500; where cut off point is Rs. 317.061

The MTSF, profit in the summers (P1), and profit in the winters (P2) graphs 8,9,10 exhibit a similar trend with failure rate *lambda* and λ_1, which means that as the failure rate rises, the MTSF and profit fall.

21. Conclusion

The significance of implementing dependability in verka milk plant is analysed and concluded upon in this study. Using the parameters laid out in tables above, it has been shown that the second model generates more money after CBM is put into effect. Results from mathematical measurements and graphs showing that MTSF and Profit drop with increasing values of failure rates must be used to gain a more in-depth understanding of the essential real influencing elements and, in turn, enhance the reliability model. But the equations derived for MTSF, assessments of the system's functionality, and profit can be used to find alternative cut-off points related to the required rates, costs, and probabilities involved. The formulas for the proposed system can then be generated by plugging in the actual numbers for the relevant rates and costs. Important decisions about the system's dependability and profitability can be made with the help of graphs showing cut-off points for key rates, costs, and revenue.

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