SURVIVAL PROBABILITY AND MEAN RESIDUAL LIFE TIMES OF SHOCK MODEL WITH ADDITIONAL RISK

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Abstract

A shock model with two types of shocks functioning in the presence of an additional risk is proposed. Survival probability and mean residual life times of the proposed models are derived and assessed through the data of life testing experiment. Model validation and estimation of survival probability and mean residual life times is done through simulation studies. Comparison of survival probabilities and mean residual life times of models functioning without and with additional risk is made.

Keywords: Damage Shock, Catastrophic Shock, Additional Risk, Survival Probability, Mean Residual Life Time, Life Testing Experiment, Maximum Likelihood Estimator.

I. Introduction

Failure of equipment/ death of a living being is usually attributed to a single cause, however various risks competing for the life of an equipment/ individual must be considered when assessing reliability/ survivability. A tool may fail due to manufacturing defect, (e.g. Geometric irregularity), not maintaining operating conditions when in use, overstressing, etc. An individual with heart failure is more likely to die from kidney failure than person without heart problem. Thus, the focus is on studying complexities of survival in the presence of competing or additional risk(s).

In our day-to-day life, we encounter with many examples wherein failure of a system/ equipment/ individual due to two types of shocks namely damage shock (causing damage) and catastrophic shock. [13] have discussed the examples of death due to heart attack (damage shock) or cardiac arrest (catastrophic shock). Here, one cannot rule out the possibility of death of a heart patient due to accident/ stroke/ renal failure.

Another example could involve an individual undergoing treatment for diabetes. Consider an individual receiving a treatment for diabetes. This marks the damage shock, where the initial impact is significant, but with proper management, the person can lead a healthy life. For the condition to lead to a more serious outcome, the damage must escalate. If the diabetes is poorly controlled, it will lead to complications such as kidney failure or severe cardiovascular issues, it happens when the damage exceeds the manageable threshold. A catastrophic shock may occur if the blood sugar level collapses suddenly due to hypoglycemia, where the person's body doesn't have enough glucose for proper functioning. This can result in loss of consciousness, and if not promptly addressed, it may lead to death. And also, additional risks come in the form of coexisting health conditions, like the development of nerve damage or an increased risk of infections due to compromised immunity. It highlights the importance of not only managing diabetes but also addressing associated risks to ensure a comprehensive approach to health and well-being.

The case of an investor who invests in a diversified portfolio of stocks also serves as an example for the problem being considered here. Consider an individual investing in a diverse portfolio of stocks. A damage shock occurs when a sudden market downturn due to economic uncertainties, has the potential to lead to a decline in the overall portfolio value. If this downturn escalates into a systemic financial crisis, exceeding the investor's tolerance threshold, it could result in a market collapse, causing significant and insurmountable losses. On the other hand, a catastrophic shock, such as an unforeseen event like a global pandemic, introduces an unpredictable element beyond routine market fluctuations and systemic crises, including the influence of geopolitical events. These events can significantly amplify challenges, contributing to the complexity of financial decision-making. An additional shock could be fluctuations in prices of other related goods. For instance, if major companies' stocks experience a decline, investors may swiftly shift their focus to alternative assets like gold or experience financial losses due to unanticipated changes in tax regulations.

Mean Residual Life (MRL) function is an interesting alternative to the survival function or the hazard function of a survival distribution. It is the expected additional lifetime given that a component has survived until time 't'. Actuaries employ MRL to design insurance portfolio. Biomedical researchers use MRL in analyzing survivorship. Increasing MRL distributions are useful models in the studies of life lengths (durations) of wars and strikes. These functions occur naturally in the studies of optimal disposal of an asset, renewal theory, dynamic programming and branching processes. MRL has been widely considered in the literature by researchers of several areas. Few of them are listed here.

A detailed analysis of the mean residual life (MRL) for various lifetime distributions, including the Weibull distribution, was studied in [15]. The mixture representations for the reliability functions of the conditional residual life and inactivity time of a coherent system with 'n' independent and identically distributed components have been derived in [11]. The modeling and inference of a family of generalized MRL models under case-cohort and nested case-control designs have been studied in [7]. The limiting process and nonparametric simultaneous confidence bands for the mean residual life function using transformation of limiting process to Brownian motion was studied by [6]. The patterns of change in life expectancy and life span equality, describing them through trajectories of mortality improvements over age and time have been explored in [2]. The developed R package 'reslife,' which enables efficient computation of mean residual lifetimes is given by [16]. Several conditions for compare the largest order statistics from resilience-scale models with reduced scale parameters in the form of mean residual life order are discussed in [5].

Here are some of the references that contribute to the literature on shock models: The fundamental work on shock models is by [1]. The reliability of a device subjected to shocks modeled by a nonhomogeneous Poisson process, demonstrating that the first-time total damage exceeds a critical threshold is an increasing failure rate average random variable was studied by [12]. A shock model framework was discussed in [4], examining scenarios where the failure rate increases over time, and the mean residual life decreases. The study in [14] investigated reliability in systems exposed to shocks from a renewal point process, offering analytical expressions for time to failure in parallel systems. The significance of analyzing product reliability through the investigation of the damage process was addressed in [8]. The classification of shock models in system reliability is discussed in [10]. The extension of generalizing the results to the generalized Polya process (GPP), where initial shocks have dependent increments, was studied in [3]. In the present study, we have further worked on the [13] paper, where the authors investigated the survival probability of a component subjected to damage and fatal (catastrophic) shocks, under fixed and random threshold setups.

In this paper, a shock model with two kinds of shocks namely damage and catastrophic shocks in the presence of an additional risk is considered. The model, its survival probability and MRL functions are discussed in Section 2. The Life Testing experiment is explained in Section 3. In Section 4, Monte-Carlo simulation is used to validate the model and mean residual life times of the models with and without additional risks are also analyzed in the same section. Discussions and conclusions are outlined in Section 5.

II. Survival Probability of the Model

Suppose a component/ system is subjected to a sequence of shocks occurring randomly in time as events of Poisson process with intensity $\lambda, \lambda > 0$. Each shock will be either a damage shock (causing damage) or catastrophic shock. If the damage exceeds the threshold of the component, the component fails or the component fails at the occurrence of catastrophic shock. The damages are non-accumulative, that is the component functions as good as new one as long as the damage does not exceed component's threshold. Let 'p' and $(1-p)$ be the probabilities that a shock is damage shock and catastrophic shock respectively. Let the damages follow exponential distribution with parameter 'θ', 'u' be the threshold of the component. The survival probability of the component at mission time 't' of the model as derived in [13] is given by

$$S_1(t) = e^{-\lambda t[1-p(1-e^{-u\theta})]} \quad (1)$$

The corresponding MRL at time 't' is given by

$$\mu_1(t) = \frac{1}{\lambda\left(1-p\left(1-e^{-u\theta}\right)\right)} \quad (2)$$

If the component is made to function under the additional risk (other than its two modes of failure) and assuming this additional risk has ageing impact. Weibull distribution (with shape parameter > 1) would be a better candidate to explain the impact of additional risk on the survival probability of the component.

Let, $S_{1A}(t)$ be the survival probability of the component which is experiencing shocks of two types as explained above and functioning under additional risk. Considering all the aforementioned features of the model, $S_{1A}(t)$ is given by

$$S_{1A}(t) = e^{-\lambda t[1-p(1-e^{-u\theta})]} . e^{-(\alpha t)^{\beta}} \quad (3)$$

The mean residual life (MRL) and other properties of several families of Weibull related life distributions are discussed in [9]. One interesting family of Weibull life distribution is with $\alpha = \frac{1}{\sqrt{2}}$ and $\beta = 2$. For this family of Weibull distribution, the survival probability and MRL are given by

$$S_A(t) = e^{-\frac{1}{2}t^2} \quad (4)$$

$$\mu_A(t) = \frac{\sqrt{2\pi}(1-\Phi(t))}{e^{-\frac{1}{2}t^2}} \tag{5}$$

From (5), it is evident that $\mu_A(t)$ has an explicit form and computationally easy.
Using this special case of Weibull in (3), the expression for $S_{1A}(t)$ reduces to

$$S_{1A}(t) = e^{-\lambda t[1-p(1-e^{-u\theta})]-\frac{t^2}{2}} \tag{6}$$

The MRL corresponding to $S_{1A}(t)$ given in (6) is given by

$$\mu_{1A}(t) = \frac{e^{\frac{1}{2}\left[\lambda\left(1-p\left(1-e^{-u\theta}\right)\right)\right]^2}.\sqrt{2\pi}\left(1-\Phi\left(t-\lambda\left(p\left(1-e^{-u\theta}\right)-1\right)\right)\right)}{e^{-\lambda t\left[1-p\left(1-e^{-u\theta}\right)\right]}.e^{-\frac{1}{2}t^2}} \tag{7}$$

The computations of $S_1(t), S_A(t)$ and $S_{1A}(t)$ for two parameter combinations $p = 0.55, \lambda = 0.40, u = 0.80, \theta = 0.65$ and $p = 0.4, \lambda = 0.70, u = 1.1, \theta = 0.55$ at various values of 't' are presented in Table 1. Also, it is to be noted that the MRL corresponding to $S_1(t)$ do not depend on 't' and are computed as 2.1833 and 1.9449 respectively for two parameter combinations considered. Table 2 presents MRL times for Weibull given in (5) and MRL times of proposed model given in (7) at different values of 't' for the two parameter combinations considered.

Table 1: *Theoretical Computation of Survival Probability*

	$p = 0.55, \lambda = 0.65, u = 1.1, \theta = 0.70$			$p = 0.45, \lambda = 0.75, u = 1.5, \theta = 0.80$		
t	$S_1(t)$	$S_A(t)$	$S_{1A}(t)$	$S_1(t)$	$S_A(t)$	$S_{1A}(t)$
0.5	0.795318	0.882497	0.701865	0.773309	0.882497	0.682443
0.75	0.709269	0.75484	0.535384	0.680032	0.75484	0.513315
1	0.63253	0.606531	0.383649	0.598007	0.606531	0.36271
1.25	0.564094	0.457833	0.258261	0.525875	0.457833	0.240763
1.5	0.503063	0.324653	0.163321	0.462444	0.324653	0.150134
1.75	0.448634	0.216265	0.097024	0.406664	0.216265	0.087947
2	0.400095	0.135335	0.054147	0.357612	0.135335	0.048398

Table 2: *Theoretical Computation of Mean Residual Life*

	$p = 0.55, \lambda = 0.65, u = 1.1, \theta = 0.70$		$p = 0.45, \lambda = 0.75, u = 1.5, \theta = 0.80$	
t	$m_A(t)$	$m_{1A}(t)$	$m_A(t)$	$m_{1A}(t)$
0.5	0.876365	0.670411	0.876365	0.650837
0.75	0.752571	0.590263	0.752571	0.574534
1	0.65568	0.525471	0.65568	0.512631
1.25	0.57843	0.472297	0.57843	0.461667
1.5	0.515816	0.428065	0.515816	0.419154
1.75	0.464307	0.390824	0.464307	0.383269
2	0.421369	0.359125	0.421369	0.352654

III. Life Testing Experiment

In order to estimate $S_{1A}(t)$ and $\mu_{1A}(t)$, suppose 'r' components with life distribution $(1 - S_{1A}(t))$ are subjected to life test. The life testing is continued until all the 'r' components fail. Let r_1, r_2 and $r_3 = (r - r_1 - r_2)$ be the numbers of components that fail due to damage shock, catastrophic shock and due to additional risk respectively. The i^{th} component fails at n_i^{th} shock and $t_{i1}, \ldots, t_{in_i}$ be the time epoch at which the i^{th} component has experienced shocks. $(t_{ij} - t_{ij-1})$ are independent exponential random variables having exponential distribution with parameter $p\lambda$, $j = 1,2, \ldots, n_i$ and $i = 1,2, \ldots, r$. It is to be noted that, the component which fails due to additional risk also experiences shocks and if any component has to fail due to additional risk, it has sustained all the damages due to damage shock and it will not experience catastrophic shock. Further it is assumed that, whenever a component fails due to damage shock (damage exceeding threshold), that damage is not measurable and the impact of catastrophic shock is also not measurable. Let X_{ij} denote the amount of damage caused by j^{th} damage shock of the i^{th} component and $X_{ij}'s$ are assumed to be independently distributed exponential random variables with parameter $\theta, \theta > 0$.

The joint distribution of $n_i, t_{i1}, t_{i2}, \ldots, t_{in_i}, X_{i1}, \ldots, X_{in_i-1}$ of the 'r_1' components that have failed due to damage shock is given by

$$\prod_{i=1}^{r_1} (p\lambda)^{n_i}\, e^{-p\lambda t_{n_i}}\; \theta^{n_i-1}\, e^{-\theta \sum_{j=1}^{n_i-1} x_{ij}}\; e^{-u\theta}$$

$$= (p\lambda)^{\sum_{i=1}^{r_1} n_i}\, e^{-p\lambda \sum_{i=1}^{r_1} t_{n_i}}\, \theta^{\sum_{i=1}^{r_1} n_i - r_1}\, e^{-\theta \sum_{i=1}^{r_1}\sum_{j=1}^{n_i-1} x_{ij}}\, e^{-r_1 u\theta} \tag{8}$$

Similarly, the joint distribution of $n_i, t_{i1}, t_{i2}, \ldots, t_{in_i}, X_{i1}, \ldots, X_{in_i-1}$ for 'r_2' components that fail due to catastrophic shock is given by

$$\prod_{i=1}^{r_2} (p\lambda)^{n_i-1}\, e^{-p\lambda t_{n_i-1}}\; \theta^{n_i-1}\, e^{-\theta \sum_{j=1}^{n_i-1} x_{ij}}\, (1-p)\lambda\, e^{-(1-p)\lambda(t_{n_i} - t_{n_i-1})}$$

$$= (p\lambda)^{\sum_{i=1}^{r_2} n_i - r_2}\;\; e^{-p\lambda \sum_{i=1}^{r_2} t_{n_i-1}}\, \theta^{\sum_{i=1}^{r_2} n_i - r_2}\, e^{-\theta \sum_{i=1}^{r_2}\sum_{j=1}^{n_i-1} x_{ij}}\, (1-p)^{r_2}\, \lambda^{r_2}\, e^{-(1-p)\lambda \sum_{i=1}^{r_2}(t_{n_i} - t_{n_i-1})} \tag{9}$$

And, letting y_i be the time epoch at which i^{th} component has failed due to additional risk, $i = 1,2, \ldots, r_3$; the joint distribution of $n_i, t_{i1}, t_{i2}, \ldots, t_{in_i}, X_{i1}, \ldots, X_{in_i}, y_i$ for 'r_3' components that fail due to additional risk is given by

$$\prod_{i=1}^{r_3} (p\lambda)^{n_i}\, e^{-p\lambda t_{n_i}}\; \theta^{n_i}\, e^{-\theta \sum_{j=1}^{n_i} x_{ij}}\; y_{ij}\, e^{-\frac{1}{2} y_{ij}^2}$$

$$= (p\lambda)^{\sum_{i=1}^{r_3} n_i}\, e^{-p\lambda \sum_{i=1}^{r_3} t_{n_i}}\, \theta^{\sum_{i=1}^{r_3} n_i}\, e^{-\theta \sum_{i=1}^{r_3}\sum_{j=1}^{n_i-1} x_{ij}}\; \prod_{i=1}^{r_3} y_{ij}\;\; e^{-\frac{1}{2}\sum_{i=1}^{r_3} y_{ij}^2} \tag{10}$$

Combining the above three cases, the joint distribution L of all the random variables involved is given by

$$L = p^{n.-r_2}\, \lambda^{n.}\, e^{-p\lambda t..}\, e^{-\lambda t.'} \theta^{n.-r_1-r_2}\, e^{-r_1 u\theta}\, e^{-\theta(x_{1.}+x_{2.}+x_{3.})}\, (1-p)^{r_2}\, y.\, e^{-\frac{1}{2} y..}\, 2^{r_3} \left(\frac{1}{\sqrt{2}}\right)^{2r_3} \tag{11}$$

where

$$t.. = \sum_{i=1}^{r_1} t_{n_i} + 2\sum_{i=1}^{r_2} t_{n_i-1} - \sum_{i=1}^{r_2} t_{n_i} + \sum_{i=1}^{r_3} t_{n_i}$$

$$t.' = \sum_{i=1}^{r_2} (t_{n_i} - t_{n_i-1})$$

$$n_i. = \sum_{i=1}^{r_i} n_i\,;\ i = 1(1)3$$

$$n. = n_1. + n_2. + n_3.$$

$y. = \prod_{i=1}^{r_3} y_{ij}$, $\qquad$ $y.. = \sum_{i=1}^{r_3} y_{ij}^2$

$x_1. = \sum_{i=1}^{r_1}\sum_{j=1}^{n_i-1} x_{ij}$, $x_2. = \sum_{i=1}^{r_2}\sum_{j=1}^{n_i-1} x_{ij}$, $x_3. = \sum_{i=1}^{r_3}\sum_{j=1}^{n_i-1} x_{ij}$

Considering L as the function of parameters, the maximum likelihood estimators $\hat{\theta}, \hat{\lambda}, \hat{p}$ respectively of θ, λ and p are given by

$$\hat{\theta} = \frac{n.-r_1-r_2}{(x_1.+x_2.+x_3.)+r_1 u} \tag{12}$$

$$\hat{\lambda} = \frac{r_2 t..+n.t.'}{t.'(t..+t.')} \tag{13}$$

$$\hat{p} = \frac{t.'(n.-r_2)}{r_2 t..+n.t.'} \tag{14}$$

Using the invariance property of MLE, the MLEs of $S_{1A}(t), \mu_{1A}(t)$ are obtained as $\hat{S}_{1A}(t)$ and $\hat{\mu}_{1A}(t)$ respectively and are given by

$$\hat{S}_{1A}(t) = e^{-\hat{\lambda}t\left[1-\hat{p}\left(1-e^{-u\hat{\theta}}\right)\right]-\frac{t^2}{2}} \tag{15}$$

$$\hat{\mu}_{1A}(t) = \frac{e^{\frac{1}{2}\left[\hat{\lambda}\left(1-\hat{p}\left(1-e^{-u\hat{\theta}}\right)\right)\right]^2}\sqrt{2\pi}\left(1-\Phi\left(t-\hat{\lambda}\left(\hat{p}\left(1-e^{-u\hat{\theta}}\right)-1\right)\right)\right)}{e^{-\hat{\lambda}t\left[1-\hat{p}\left(1-e^{-u\hat{\theta}}\right)\right]}\, e^{-\frac{1}{2}t^2}} \tag{16}$$

IV. Simulation Study and Analysis

Monte-Carlo simulation is used to generate the random variables of the model. For considered values of $u = u_0, p = p_0, \theta = \theta_0, \lambda = \lambda_0$ using the following algorithm, all the random variables involved are generated.

Step 1: Generate a random number w_i from $U(0,1)$. If $0 < w_i < \left(1 - e^{-\frac{t^2}{2}}\right)$, then it is considered that the failure of component is due to additional risk. In this case;

i. Initialize $n_i, t_i.$ and $x_i.$ with zero.

ii. Generate y_i Weibull random variable with $\sigma = \frac{1}{\sqrt{2}}$, $\beta = 2$.

iii. Generate t_{i1} with exp $(p_0\lambda_0)$.

iv. Generate x_{i1}, an exp(θ_0) random variable.

v. Compare t_{i1} with y_i and x_{i1} with u_0 .

vi. If $(t_{i1} < y_i)$ and $(x_{i1} < u_0)$, then n_i is incremented by 1 and t_{i1} is added to $t_{i.}$, x_i is added to $x_{i.}$.

Steps (ii) to (vi) are repeated until either $x_{i1} > u$ or $t_{i1} > y_i$.

Step 2: If $w_i \geq e^{-\frac{1}{2}t^2}$, the failure of the component is attributed to either damage shock or catastrophic shock.

i. A uniform random variable $U(0,1)$ 'V_i' is generated. If $0 < V_i < p = p_0$, then the failure of the component is due to damage shock.

ii. An exp (θ_0) random variable X_{i1} is generated, n_i is raised by 1. If $X_{i1} < u_0$, this step is repeated. The process is stopped when it is found that $X_{i1} > u_0$.

iii. n_i number of exp $(p_0\lambda_0)$ (inter-arrival times) are generated and are added to get t_{in_i} .

In this way the random variables $n_i, X_{i1}, \ldots, X_{in_i-1}, t_{in_i}$ are generated.

On the other hand, if $V_i \geq (p = p_0)$, the failure of component is due to catastrophic shock. The random variables $n_i, X_{i1}, \ldots, X_{in_i-1}$ are generated as in Step 2(ii). $(n_i - 1)$ exponential random variables with parameter $p_0\lambda_0$ are generated, which will be inter-arrival times. Adding these inter-arrival times t_{in_i-1} is obtained. Another exponential random variable with parameter $(1-p_0)\lambda_0$ is generated which will be $(t_{in_i} - t_{in_i-1})$.

Steps 1 and 2 are repeated for $r = 25,30,40,50,100$ and the statistics $n., t.., t.', y., y.., x_1., x_2.$ and $x_{3.}$

are computed using which the MLEs of parameters are obtained. By using these MLEs of parameters in the expressions for $S_{1A}(t), \mu_{1A}(t)$, $\hat{S}_{1A}(t)$, $\hat{\mu}_{1A}(t)$ are obtained for $t = 0.5, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00$.

The whole process is repeated for $M = 10000$ times. The means of the estimated $S_{1A}(t)$ and $\mu_{1A}(t)$ along with their mean absolute biases **(bold figures)** for the parameter combination $p = 0.55, \lambda = 0.65, u = 1.1, \theta = 0.70$ with 10,000 repetitions are presented in Tables 3 and 4 respectively. Tables 5 and 6 provide the same results for $p = 0.45, \lambda = 0.75, u = 1.5, \theta = 0.80$.

Table 3: *Estimated $S_{1A}(t)$ and its Mean Absolute Bias for $p = 0.55, \lambda = 0.65, u = 1.1, \theta = 0.70$*

		$S_{1A}(t)$ **Estimated**				
t	$S_{1A}(t)$	r = 25	r = 30	r = 40	r = 50	r = 100
0.5	0.701865	0.63938 **0.062486**	0.656183 **0.045683**	0.660343 **0.041523**	0.679049 **0.022817**	0.68783 **0.014035**
0.75	0.535384	0.470406 **0.064979**	0.483974 **0.05141**	0.488583 **0.046801**	0.509491 **0.025894**	0.519406 **0.015979**
1	0.383649	0.318379 **0.06527**	0.335333 **0.048316**	0.339598 **0.044051**	0.359111 **0.024538**	0.368459 **0.015191**
1.25	0.258261	0.204561 **0.053701**	0.218266 **0.039995**	0.221742 **0.036519**	0.237781 **0.02048**	0.245543 **0.012718**
1.5	0.163321	0.123468 **0.039852**	0.133461 **0.02986**	0.136015 **0.027305**	0.147905 **0.015416**	0.153717 **0.009603**
1.75	0.097024	0.070008 **0.027016**	0.076662 **0.020362**	0.078376 **0.018648**	0.086426 **0.010598**	0.090401 **0.006623**
2	0.054147	0.03729 **0.016857**	0.041367 **0.01278**	0.042426 **0.011721**	0.047442 **0.006705**	0.049944 **0.004203**

Table 4: *Estimated $\mu_{1A}(t)$ and its Mean Absolute Bias for $p = 0.55, \lambda = 0.65, u = 1.1, \theta = 0.70$*

		$\mu_{1A}(t)$ **Estimated**				
t	$\mu_{1A}(t)$	r = 25	r = 30	r = 40	r = 50	r = 100
0.5	0.670411	0.574976 **0.095435**	0.619012 **0.051398**	0.621026 **0.049384**	0.63513 **0.035281**	0.651381 **0.019029**
0.75	0.590263	0.512992 **0.077271**	0.548831 **0.041432**	0.550462 **0.039801**	0.561868 **0.028395**	0.574971 **0.015292**
1	0.525471	0.461967 **0.063504**	0.491556 **0.033915**	0.492897 **0.032574**	0.50226 **0.023211**	0.512989 **0.012482**
1.25	0.472297	0.419406 **0.052891**	0.444151 **0.028146**	0.445268 **0.027029**	0.453059 **0.019238**	0.461964 **0.010333**
1.5	0.428065	0.383482 **0.044583**	0.404417 **0.023649**	0.405359 **0.022707**	0.41192 **0.016146**	0.419403 **0.008662**
1.75	0.390824	0.352837 **0.037987**	0.370733 **0.020091**	0.371536 **0.019288**	0.377121 **0.013703**	0.38348 **0.007344**
2	0.359125	0.326443 **0.032683**	0.341885 **0.01724**	0.342576 **0.016549**	0.347378 **0.011747**	0.352835 **0.00629**

Table 5: *Estimated $S_{1A}(t)$ and its Mean Absolute Bias for $p = 0.45, \lambda = 0.75, u = 1.5, \theta = 0.80$*

		$S_{1A}(t)$ Estimated				
t	$S_{1A}(t)$	r = 25	r = 30	r = 40	r = 50	r = 100
0.5	0.682443	0.611759 **0.070683**	0.614114 **0.068329**	0.626848 **0.05559481**	0.633057 **0.049385**	0.639267 **0.043175**
0.75	0.513315	0.435668 **0.077647**	0.438186 **0.075129**	0.451885 **0.061430**	0.458616 **0.054699**	0.465381 **0.047934**
1	0.36271	0.291466 **0.071243**	0.293714 **0.068995**	0.306021 **0.056688**	0.312114 **0.050596**	0.318267 **0.044442**
1.25	0.240763	0.183179 **0.057584**	0.184947 **0.055816**	0.194684 **0.046079**	0.199541 **0.041222**	0.204471 **0.036292**
1.5	0.150134	0.108149 **0.041985**	0.109402 **0.040731**	0.11635 **0.033783**	0.119842 **0.030291**	0.123403 **0.026730**
1.75	0.087947	0.059982 **0.027965**	0.060794 **0.027153**	0.065322 **0.022625**	0.067615 **0.020332**	0.069965 **0.017982**
2	0.048398	0.031252 **0.017145**	0.031736 **0.016661**	0.034451 **0.013946**	0.035837 **0.012560**	0.037264 **0.011133**

Table 6: *Estimated $\mu_{1A}(t)$ and its Mean Absolute Bias for $p = 0.45, \lambda = 0.75, u = 1.5, \theta = 0.80$*

		$\mu_{1A}(t)$ Estimated				
t	$\mu_{1A}(t)$	r = 25	r = 30	r = 40	r = 50	r = 100
0.5	0.650837	0.606371 **0.044466**	0.608328 **0.04251**	0.618495 **0.032342**	0.619872 **0.030965**	0.623134 **0.027704**
0.75	0.574534	0.538577 **0.035957**	0.540165 **0.034369**	0.548412 **0.026122**	0.549528 **0.025006**	0.552169 **0.022365**
1	0.512631	0.483114 **0.029517**	0.484423 **0.028208**	0.491211 **0.02142**	0.492129 **0.020502**	0.494299 **0.018332**
1.25	0.461667	0.43711 **0.024558**	0.438202 **0.023465**	0.443864 **0.017804**	0.444628 **0.017039**	0.446436 **0.015232**
1.5	0.419154	0.398474 **0.020681**	0.399397 **0.019758**	0.404175 **0.01498**	0.404819 **0.014335**	0.406343 **0.012811**
1.75	0.383269	0.365663 **0.017605**	0.366451 **0.016817**	0.370526 **0.012742**	0.371076 **0.012193**	0.372374 **0.010894**
2	0.352654	0.337519 **0.015135**	0.338198 **0.014456**	0.341708 **0.010946**	0.34218 **0.010473**	0.343298 **0.009356**

V. Results and Conclusion

From Tables 1 and 2, it is found that, for both parameter combinations, the theoretical values of $S_1(t)$, $S_A(t)$ and $S_{1A}(t)$ and $\mu_1(t)$, $\mu_A(t)$ and $\mu_{1A}(t)$ are non-increasing in 't'. $\mu_1(t)$ is independent of time 't', so its values for any considered parameter combinations will be constant for all values of 't'. The model functioning in the presence of additional risk has smaller survival probability and mean residual life times. From tables 3 and 5, it is clear that the Maximum Likelihood Estimators (MLEs) underestimate the true survival probability. The estimated survival probability for all time points (t) tend to improve as the sample size increases at all time points. Also, mean absolute bias **(bold figures)** decreases as the sample size increases, implying that larger samples lead to more accurate estimators, which is a desirable statistical property. Tables 4 and 6 collectively substantiate the inference drawn regarding the mean residual life times, akin to the analysis conducted for survival probability.

To improve the performance of Maximum Likelihood Estimators (MLEs), one can think of greater sample size. Increase in sample size may not be a better choice, especially when one is dealing with real life cases and/ or high-cost units. Alternatively, one can explore other methods of estimation.

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