ENHANCING REDUNDANT SYSTEM PERFORMANCE: A STOCHASTIC MODEL FOR OPTIMIZED INSPECTION STRATEGIES POST-FAILURE

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Abstract

This paper delves into the strategic utilization of inspections to determine the appropriate action for components within redundant systems following unit and switch failures. Post-failure, the timely execution of repair and replacement procedures is paramount for restoring system functionality. By assigning inspection tasks to servers, this paper aims to evaluate the condition of system components and make informed decisions regarding repair or replacement. It addresses the standardization of inspection processes and subsequent repair/replacement protocols for industrial systems encountering failures. Introducing a model, the study endeavors to bolster system reliability and availability by addressing failures caused by faults through inspection and subsequent repair/replacement actions. Employing a quantitative approach, it provides insights into maintaining system reliability and availability via a stochastic framework. By integrating unit and switch inspections into the analysis, the paper proposes a strategic approach to optimizing redundant system operations, facilitating effective decision-making concerning repair and replacement strategies post-failure.

Keywords: switch, server, inspection, replacement, and repair.

1. Introduction

Ensuring quality, dependability, and efficiency is paramount in today's industrial landscape; spanning sectors like manufacturing, aerospace, and automotive, where meeting precision standards and exceeding customer expectations are non-negotiable. Central to achieving these goals is the implementation of efficient inspection methods within industrial systems. Inspection procedures are indispensable for defect detection, ensuring product integrity, and upholding safety standards. The evaluation of the operational status is facilitated through inspections. Standby systems typically employ two approaches to inspection policies: routine checks or inspections triggered by failures. Periodic inspection policies aim to optimize standby system performance by conducting inspections at predetermined intervals [1]. Timing periodic inspections optimally can enhance standby system dependability while minimizing costs [2]. Alternatively, the expense and downtime associated with directly repairing a failed unit can be mitigated through post-failure inspections [3]. When a unit malfunctions, the options of repair or replacement are considered based on its operational state [4]. Replacement involves substituting a defective unit with a new one, which can be reused after the switch [5]. The responsibility for all repair and replacement tasks lies with a single server, emphasizing the critical role of skilled personnel in maintaining system reliability. However, over time, server degradation may occur, leading to increased downtime and necessitating updates [6].

In the event of a main unit failure in a backup system, the switch plays a crucial role in activating the standby unit to ensure system availability and reliability. Switching may occur at predetermined intervals or when the operational unit fails [7]. Failures during switching can affect system functionality, necessitating reboots and repairs [8] and [9]. Switching reliability varies, with standby systems more prone to faulty switches [10]. Post-switch failure inspections dictate repair or replacement based on utility [11]. Insufficient switching can compromise warm standby system reliability, requiring intervention from repair personnel [12]. Server or switch failures during tasks can degrade system performance and significantly affect expenses [13] and [14]. Probabilistic models offer insights into system performance, availability, and failure detection [15]. Maintaining high availability hinges on ensuring the reliability and efficiency of switches and servers, with probabilistic models providing the most accurate assessment of standby system profitability. The Weibull distribution is particularly suitable for simulating random failures and evaluating standby system profitability [16]. Additionally, post-failure economic evaluations can be conducted for switches [17].

This paper introduces a probabilistic model of a standby system comprising two identical units, one serving as the primary operating unit and the other as the cold standby unit. In the event of the main unit failure, the switch activates the cold standby. Subsequently, the server initiates an inspection of the unit to determine whether repair or replacement is warranted, following similar procedures for switches after failure. Replacement is preferred only when repair costs are deemed prohibitive. The server oversees all inspection, repair, and replacement tasks, but can only handle one task at a time. Repairing switches takes time, whereas replacement is expedient. Numerical simulations in this study follow the Weibull distribution for accuracy.

2. Notations

O	The unit is in operative mode.
Cs	The unit is kept as cold standby.
St	The switching mechanism is good.
Se	The server is good.
Csw	The cold standby unit is under waiting.
p/q	The switch is under operation/failed.
F_{ur} / F_{UR}	The unit is under repair/under repair continuously from previous state.
F_{wr} / F_{WR}	The failed unit is waiting for repair/waiting for repair continuously from previous state.
St_{ur} / St_{UR}	The switch mechanism is under repair/under repair continuously from previous state.
St_{wr} / St_{WR}	The switch mechanism is failed and waiting for repair/under treatment continuously from previous state.
St_{wi} / St_{WI}	The switch mechanism is waiting for inspection /continuously waiting for inspection from previous state.
Se_{ut} /Se_{UT}	The server is failed and under treatment/under treatment continuously from previous state.

z(t)/Z(t)	pdf/cdf of failure rate of the unit.
r(t)/R(t)	pdf/cdf of failure rate of the server.
f(t)/F(t)	pdf/cdf of repair time of the failed unit.
h(t)/H(t)	pdf/cdf of repair time of the failed switch.
n(t)/N(t)	pdf/cdf of switch inspection time of the server.
m(t)/M(t)	pdf/cdf of unit inspection time of the server.
s(t)/S(t)	pdf/cdf of the treatment time of the server.
c/d	Probability of Switch repair /replacement feasibility after inspection.
a/b	Probability of Unit repair /replacement feasibility after inspection.
$M_i(t)$	Probability that the system is up initially in state Si ∈ E and remains up at time t without visiting any other regenerative state.
$W_i(t)$	Probability that the unit, switch, and server remain busy in state Si up to time 't' without transitioning to another regenerative state or returning to the same state via non-regenerative states.
○/ □	Representation of regenerative states and failed regenerative states in a diagram.
●	Regenerative points.

3. Development of Model

3.1 Assumptions

- One unit is initially powered on and the other is placed in cold standby.
- The unit is repaired directly after a failure, but the switch is checked to see if it can be repaired/replaced.
- Switch switching is instantaneous.
- If the main unit fails then it goes under the inspection process, to check the feasibility of its repair or replacement.
- If switch fails then it goes under the inspection process, to check the feasibility of its repair or replacement.
- The switch is prioritized for repair or replacement after inspection and failure.
- The server can fail while doing its job, but not in an idle state.
- After a failure, the server moves directly to the recovery phase.
- Replacement is instantaneous.
- Inspection/ repair priority is given to switch after failure.
- All repairs and treatments are perfect.
- Random variables are statistically independent.

3.2 States of the System

The following are possible transition states of the system model.

The regenerative states:

$S_0=(O,Cs,St,Sv), S_1=(F_{ui},O), S_2=(F_{ur},O),\ S_3=(O,F_{wi},Sv_{ut}), S_4=(O,F_{wr},Sv_{ut}).$

The failed regenerative states:

$S_5=(F_{wi},Cs_w,St_{ui}),$

The non-regenerative states:

$S_6=\left(F_{WI},Cs_w,St_{wi},Sv_{ut}\right),S_7=\left(F_{UI},F_{wi}\right),S_8=\left(F_{wi},F_{WI},Sv_{ut}\right),$

$S_9=\left(F_{wi},F_{WR},Sv_{UT}\right),S_{10}=\left(F_{UR},F_{wi}\right),S_{11}=\left(F_{wi},F_{WI},Sv_{UT}\right),$

$S_{12}=\left(F_{WI},Cs_w,St_{ui}\right),S_{13}=\left(F_{WI},Cs_w,St_{ur}\right),S_{14}=\left(F_{UI},F_{wi}\right),$

$S_{15}=\left(F_{WI},Cs_w,St_{wr},Sv_{ut}\right),S_{16}=\left(F_{ur},F_{WI}\right),S_{17}=\left(F_{wr},F_{WI},Sv_{ut}\right).$

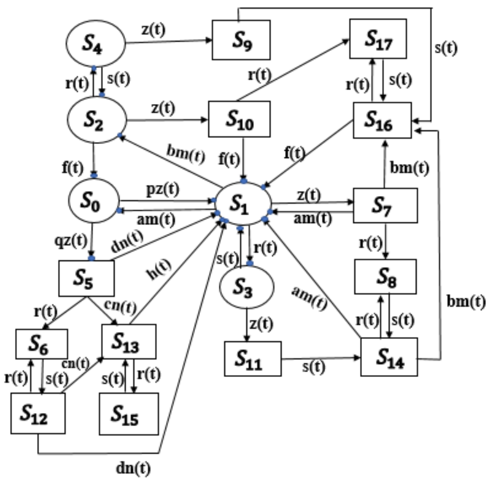

Figure 1: *State transition diagram of model*

3.3 Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero element

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \tag{1}$$

The Mean Sojourn time μ_i in state S_i are given by:

$$\mu_i = E(t) = \int_0^\infty P(T > t)dt \tag{2}$$

We get

$$p_{0,1}=\int_0^\infty pz(t)dt,\ p_{0,5}=\int_0^\infty qz(t)dt,\ p_{1,0}=\int_0^\infty am(t)\bar{Z}(t)\bar{R}(t)dt,$$

$$p_{1,2}=\int_0^\infty bm(t)\bar{Z}(t)\bar{R}(t)dt,\ p_{1,3}=\int_0^\infty r(t)\bar{M}(t)\bar{Z}(t)dt,\ p_{1,4}=\int_0^\infty z(t)\bar{R}(t)\bar{M}(t)dt,$$

$$p_{2,0}=\int_0^\infty f(t)\bar{Z}(t)\bar{R}(t)dt,\ p_{2,4}=\int_0^\infty r(t)\bar{F}(t)\bar{Z}(t)dt,\ p_{2,10}=\int_0^\infty z(t)\bar{R}(t)\bar{F}(t)dt,$$

$$p_{3,1}=\int_0^\infty s(t)\bar{Z}(t)dt,\ p_{3,11}=\int_0^\infty z(t)\bar{S}(t)dt,\ p_{4,2}=\int_0^\infty s(t)\bar{Z}(t)dt,$$

$$p_{4,9}=\int_0^\infty z(t)\bar{S}(t)dt,\ p_{5,1}=\int_0^\infty dn(t)\bar{R}(t)dt,\ p_{5,6}=\int_0^\infty r(t)\bar{N}(t)dt,$$

$$p_{5,13}=\int_0^\infty cn(t)\overline{R}(t)dt,\ p_{6,12}=\int_0^\infty s(t)dt,\ p_{7,1}=\int_0^\infty am(t)\overline{R}(t)dt,$$

$$p_{7,8}=\int_0^\infty r(t)\overline{M}(t)dt,\ p_{7,16}=\int_0^\infty bm(t)\overline{R}(t)dt,\ p_{8,14}=\int_0^\infty s(t)dt,$$

$$p_{9,16}=\int_0^\infty s(t)dt,\ p_{10,1}=\int_0^\infty f(t)\overline{R}(t)dt,\ p_{10,17}=\int_0^\infty r(t)\overline{F}(t)dt,$$

$$p_{11,14}=\int_0^\infty s(t)dt,\ p_{12,1}=\int_0^\infty dn(t)\overline{R}(t)dt,\ p_{12,6}=\int_0^\infty r(t)\overline{N}(t)dt,$$

$$p_{12,13}=\int_0^\infty cn(t)\overline{R}(t)dt,\ p_{13,1}=\int_0^\infty h(t)\overline{R}(t)dt,\ p_{13,15}=\int_0^\infty r(t)\overline{H}(t)dt,$$

$$p_{14,1}=\int_0^\infty am(t)\overline{R}(t)dt,\ p_{14,8}=\int_0^\infty r(t)\overline{M}(t)dt,\ p_{14,16}=\int_0^\infty bm(t)\overline{R}(t)dt,$$

$$p_{15,13}=\int_0^\infty s(t)dt,\ p_{16,1}=\int_0^\infty f(t)\overline{R}(t)dt,\ p_{16,17}=\int_0^\infty r(t)\overline{F}(t)dt,$$

$$p_{17,16}=\int_0^\infty s(t)dt,$$

The expressions for mean sojourn times are as follows:

$$\mu_0=\int_0^\infty \overline{Z}(t)dt,\ \mu_1=\int_0^\infty \overline{Z}(t)\overline{R}(t)\overline{M}(t)dt,\ \mu_2=\int_0^\infty \overline{Z}(t)\overline{F}(t)dt,\ \mu_3=\int_0^\infty \overline{Z}(t)\overline{S}(t)dt,\ \mu_4=\int_0^\infty \overline{Z}(t)\overline{S}(t)dt.$$

4. System's Performance Measures

4.1 Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the c.d.f of the first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$$\phi_i(t)=\sum_j Q_{i,j}(t)[s]\phi_j(t)+\sum_k Q_{i,k}(t) \qquad i=0,1,3,4,5 \tag{3}$$

Taking LST of Eq. (3) and solving for $\tilde{\phi}_0(s)$, we have

$$R^*(s)=\frac{1-\tilde{\phi}_0(s)}{s} \tag{4}$$

The reliability R(t) can be obtained by taking inverse Laplace transition of Eq.(4) and MTSF is given by

$$MTSF(t)=\lim_{s\to 0}R^*(s)=\lim_{s\to 0}\frac{1-\tilde{\phi}_0(s)}{s}=\frac{N_1}{D_1} \tag{5}$$

Where

$$N_1=\mu_0[1-p_{1,3}p_{3,1}][1-p_{2,4}p_{4,2}]+p_{0,1}\left[\mu_1+p_{1,3}\mu_3\right]+p_{0,1}p_{1,2}\left[\mu_2+p_{2,4}\mu_4\right]$$

$$D_1=[1-p_{1,3}p_{3,1}-p_{1,0}p_{0,1}][1-p_{2,4}p_{4,2}]-p_{2,0}p_{1,2}p_{0,1}.$$

4.2 Steady State Availability

$M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0=\int_0^\infty \bar{Z}(t)dt,\ M_1=\int_0^\infty \bar{Z}(t)\bar{R}(t)\bar{M}(t)dt,\ M_2=\int_0^\infty \bar{Z}(t)\bar{F}(t)dt,\ M_3=\int_0^\infty \bar{Z}(t)\bar{S}(t)dt,\ M_4=\int_0^\infty \bar{Z}(t)\bar{S}(t)dt,$$

Let $A_i(t)$ be the probability that the system is in up-state at an instant 't' given that the system entered regenerative state S_i at t=0. The recursive relations for $A_i(t)$ are as follows:

$$A_i(t)=M_i(t)+\sum_j q_{i,j}^{(n)}(t)[c]A_j(t) \qquad i=0,1,2,3,4,5 \tag{6}$$

Where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. Taking LT of Eq. (6) and solving, the steady state availability is given by

$$A_0=\lim_{s\to 0} sA_0^*(s)=\frac{N_2}{D_2} \tag{7}$$

Where

$$N_2=(p_{1,0}\mu_0+\mu_1+p_{1,3}\mu_3)(1-p_{2,4}p_{4,2})+p_{1,2}(p_{2,0}\mu_0+\mu_2+p_{2,4}\mu_4)$$

$$D_2=(p_{1,0}\mu_0+\mu_1^{'}+p_{1,3}\mu_3^{'}+p_{0,5}\mu_5^{'})(1-p_{2,4}p_{4,2})+p_{1,2}(p_{2,0}\mu_0+\mu_2^{'}+p_{2,4}\mu_4^{'})\cdot$$

4.3 Busy Period Analysis for Server

4.3.1 Due to Inspection

Let $B_i^I(t)$ be the probability that the server is busy in inspection at an instant t given that the system entered regenerative state S_i at t = 0. The recursive relations for $B_i^I(t)$ are as follows:

$$B_i^I(t)=W_i^I(t)+\sum_j q_{i,j}^{(n)}(t)[c]B_j^I(t) \qquad i=0,1,2,3,4,5 \tag{8}$$

Where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. $W_i^I(t)$ be the probability that the server is busy in state S_i due to repair of the unit up to time 't' without making any transition to any other regenerative state or returning to the same via one or more non-regenerative state

$$W_1^I(t)=\bar{Z}(t)\bar{M}(t)\bar{R}(t)+(z(t)\bar{R}(t)[c]1)\bar{M}(t)+(z(t)\bar{M}(t)\bar{R}(t)[c]r(t)\bar{M}(t)[c]1)\bar{S}(t)+(z(t)\bar{M}(t)\bar{R}(t)[c]r(t)\bar{M}(t)[c]s(t)[c]1)M(t)$$

$$W_5^I(t)=\bar{N}(t)\bar{R}(t)+(r(t)\bar{N}(t)[c]1)\bar{S}(t)+(r(t)\bar{N}(t)[c]s(t)[c]1)\bar{N}(t)+(cn(t)\bar{R}(t)[c]1)\bar{H}(t)\bar{R}(t)+(cn(t)\bar{R}(t)[c]r(t)\bar{H}(t)[c]1)\bar{S}(t)+(cn(t)\bar{R}(t)[c]r(t)\bar{H}(t)[c]s(t)[c]1)\bar{H}(t)$$

Using LT, of Eq. (8) and solving for $B_0^{I*}(s)$, we have

$$B_0^I=\lim_{s\to 0} sB_0^{I*}(s)=\frac{N_3^I}{D_3^I} \tag{9}$$

Where

$$N_{13}^I=(1-p_{2,4}p_{4,2})W_1^I(0)+p_{0,5}(p_{1,0}(1-p_{2,4}p_{4,2})+p_{1,2}p_{2,0})W_5^I(0)$$

$$D_3^I=(1-p_{0,4}(p_{4,0}+p_{4,5}p_{5,0}))(\mu_1^{'}+\mu_3^{'}p_{1,3})+p_{1,0}(\mu_0+p_{0,2}\mu_2^{'}+p_{0,4}\mu_4^{'}+\mu_5^{'}p_{4,5})\cdot$$

4.3.2 Due to Repair

Let $B_i^R(t)$ be the probability that the server is busy in repair of the switch or unit due to inspection at an instant 't' given that the system entered state S_i at time t=0. The recursive relations for $B_i^R(t)$ are as follows:

$$B_i^R(t)=W_i^R(t)+\sum_j q_{i,j}^{(n)}(t)[c]B_j^R(t) \qquad i=0,1,2,3,4,5 \qquad (10)$$

Where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. $W_i^R(t)$ be the probability that the server is busy in state S_i due to preventive maintenance of server up to time 't' without making any transition to any other regenerative state or returning to the same via one or more non-regenerative state

$W_2^R(t)=\bar{Z}(t)\bar{F}(t)\bar{R}(t)+(z(t)\bar{R}(t)[c]1)\bar{F}(t)+(z(t)\bar{F}(t)\bar{R}(t)[c]r(t)\bar{F}(t)[c]1)\bar{S}(t)+(z(t)\bar{F}(t)\bar{R}(t)[c]r(t)\bar{F}(t)[c]s(t)[c]1)\bar{F}(t)$

Using LT, of Eq. (10) and solving for $B_0^{R*}(s)$, we have

$$B_0^R=\lim_{s\to 0} sB_0^{R*}(s)=\frac{N_3^R}{D_3^R} \qquad (11)$$

Where

$N_3^R = p_{12}W_2^{R*}(0)$,

$D_3^R = (1-p_{0,4}(p_{4,0}+p_{4,5}p_{5,0}))(\mu_1'+\mu_3'p_{1,3})+p_{1,0}(\mu_0+p_{0,2}\mu_2'+p_{0,4}\mu_4'+\mu_5'p_{4,5})$.

4.4 Expected Number of Treatment of the Server

Let $T_i(t)$ be the expected number of treatment of the failed server in (0, t] given that the system entered the regenerative state S_i at t=0. The recursive relations for $T_i(t)$ are given as:

$$T_i(t)=\sum_j Q_{i,j}(t)[s][\varphi_j+T_j(t)] \qquad i=0,1,2,3,4,5 \qquad (12)$$

where

$$\varphi_j=\begin{cases}1 & \text{if the server performs the task in state } S_j.\\ 0 & \text{otherwise}\end{cases}$$

Using LT, of Eq. (12) and solving for $\tilde{T}_0(s)$,we get

$$T_0=\lim_{s\to 0} s\tilde{T}_0(s)=\frac{N_5}{D_5} \qquad (13)$$

Where

$N_4 = p_{1,3}p_{3,1}(1-p_{2,4}p_{4,2})+p_{1,2}p_{2,4}p_{4,2}$

$D_4 = (1-p_{0,4}(p_{4,0}+p_{4,5}p_{5,0}))(\mu_1'+\mu_3'p_{1,3})+p_{1,0}(\mu_0+p_{0,2}\mu_2'+p_{0,4}\mu_4'+\mu_5'p_{4,5})$.

4.5 Expected Number of Inspections

Let $I_i^m(t)$ be the expected number of inspections of the unit and switch in (0, t] given that the system entered regenerative state S_i at time t=0. The recursive relations for $I_i^m(t)$ are as follows:

$$I_i^m(t)=\sum_j Q_{i,j}(t)[s][\varphi_j+I_j^m(t)] \qquad i=0,1,2,3,4,5 \qquad (14)$$

where

$$\varphi_j=\begin{cases}1 & \text{if the server performs the task in state } S_j.\\ 0 & \text{otherwise}\end{cases}$$

Using LT, of Eq. (14) and solving for $\tilde{I}_0^m(s)$,we get

$$I_0^m=\lim_{s\to 0} s\tilde{I}_o^m(s)=\frac{N_k}{D_k} \qquad (15)$$

Expected number of unit inspections $I_i^u(t)$ when m=u and k=5 in Eq. (15), we get

$N_5 = p_{1,2}p_{0,5}p_{2,0}+(1+p_{1,0}p_{0,5})(1-p_{2,4}p_{4,2})$,

$D_5 = (1-p_{0,4}(p_{4,0}+p_{4,5}p_{5,0}))(\mu_1'+\mu_3'p_{1,3})+p_{1,0}(\mu_0+p_{0,2}\mu_2'+p_{0,4}\mu_4'+\mu_5'p_{4,5})$.

Expected number of switch inspections $I_i^s(t)$ when m=s and k=6 in Eq. (15), we get

$N_6 = p_{1,2}p_{0,5}p_{2,0} + p_{1,0}p_{0,5}(1-p_{2,4}p_{4,2})$,

$D_6 = (1- p_{0,4}(p_{4,0}+p_{4,5}p_{5,0}))(\mu_1^{'}+\mu_3^{'}p_{1,3})+ p_{1,0}(\mu_0+p_{0,2}\mu_2^{'}+p_{0,4}\mu_4^{'}+\mu_5^{'}p_{4,5})$.

4.6 Expected Number of Replacements

Let $P_i^m(t)$ be the expected number of replacements of unit and switch in (0, t] given that the system entered the regenerative state S_i at t=0. The recursive relations for $P_i^m(t)$ are as follows:

$$P_i^m(t)=\sum_j Q_{i,j}(t)[s][\varphi_j+P_i^m(t)] \qquad i=0,1,2,3,4,5 \tag{16}$$

Where

$$\varphi_j=\begin{cases}1 & \text{if the server performs the task in state } S_j.\\ 0 & \text{otherwise}\end{cases}$$

Using LT, of Eq. (16) and solving for $\tilde{P}_0^m(s)$,we get

$$P_0^m=\lim_{s\to 0} s\tilde{P}_0^m(s)=\frac{N_k}{D_k} \tag{17}$$

Expected number of unit replacements $\tilde{P}_0^u(s)$ when m=u and k=7 in Eq. (17), we get

$N_7 = (1-p_{2,4}p_{4,2})(p_{1,0}+p_{1,1.7}+p_{1,1.7,(8,14)^n}+p_{1,3}(p_{3,1.11,14}+p_{3,1.11,(14,8)^n}))$,

$D_7 = (1- p_{0,4}(p_{4,0}+p_{4,5}p_{5,0}))(\mu_1^{'}+\mu_3^{'}p_{1,3})+ p_{1,0}(\mu_0+p_{0,2}\mu_2^{'}+p_{0,4}\mu_4^{'}+\mu_5^{'}p_{4,5})$

Expected number of switch replacements $\tilde{P}_0^s(s)$ when m=s and k=8 in Eq. (17), we get

$N_8 = p_{0,5}\{p_{1,0}(1-p_{2,4}p_{4,2})+p_{2,0}p_{1,2}\}(p_{5,1}+p_{5,1.(6,12)^n})$,

$D_8 = (1- p_{0,4}(p_{4,0}+p_{4,5}p_{5,0}))(\mu_1^{'}+\mu_3^{'}p_{1,3})+ p_{1,0}(\mu_0+p_{0,2}\mu_2^{'}+p_{0,4}\mu_4^{'}+\mu_5^{'}p_{4,5})$

4.7 Expected Number of Repairs

Let $O_i^m(t)$ be the expected number of repairs of the unit or switch by the server in (0, t] given that the system entered regenerative state S_i at time t=0. The recursive relations for $O_i^m(t)$ are as follows:

$$O_i^m(t)=\sum_j Q_{i,j}(t)[s][\varphi_j+O_j^m(t)] \qquad i=0,1,2,3,4,5 \tag{18}$$

Where

$$\varphi_j=\begin{cases}1 & \text{if the server performs the task in state } S_j.\\ 0 & \text{otherwise}\end{cases}$$

Using LT, of Eq. (18) and solving for $\tilde{O}_0^m(s)$,we get

$$O_0^m=\lim_{s\to 0} s\tilde{O}_0^m(s)=\frac{N_k}{D_k} \tag{19}$$

Expected number of unit repairs $\tilde{O}_0^u(s)$ when m=u and k=9 in Eq. (19), we get

$N_9 =(1- p_{1,0}-p_{1,1.7}-p_{1,1.7,(8,14)^n} - p_{1,3}(p_{3,1}+p_{3,1.11,14}+p_{3,1.11,(14,8)^n}))(1- p_{2,4}p_{4,2})$,

$D_9 = (1- p_{0,4}(p_{4,0}+p_{4,5}p_{5,0}))(\mu_1^{'}+\mu_3^{'}p_{1,3})+ p_{1,0}(\mu_0+p_{0,2}\mu_2^{'}+p_{0,4}\mu_4^{'}+\mu_5^{'}p_{4,5})$

Expected number of switch repairs $\tilde{O}_0^m(s)$ when m=s and k=10 in Eq. (19), we get

$N_{10} = p_{0,5}\{p_{1,0}(1-p_{2,4}p_{4,2})+p_{2,0}p_{1,2}\}(1- p_{5,1}- p_{5,1.(6,12)^n})$,

$D_{10} = (1- p_{0,4}(p_{4,0}+p_{4,5}p_{5,0}))(\mu_1^{'}+\mu_3^{'}p_{1,3})+ p_{1,0}(\mu_0+p_{0,2}\mu_2^{'}+p_{0,4}\mu_4^{'}+\mu_5^{'}p_{4,5})$

5. The Profit

The Profit incurred to the system model in steady state is obtained after the deduction of all expenses from the total revenue generated from all sources.

$$P(t) = C_0\, A_0(t) - \sum_{j=1}^{9} C_j L_j(t) \tag{20}$$

Where

j :	1	2	3	4	5	6	7	8	9
$L_j(t)$:	B_0^I	B_0^R	T_0	I_0^u	I_0^s	P_0^u	P_0^s	O_0^u	O_0^s

Where

C_0 = Revenue per unit up-time of the system

C_1 = Cost per unit time for which server is in inspection

C_2 = Cost per unit time for which server is busy in repair of failed unit or switch

C_3 = Cost per treatment of the server

C_4 = Cost per unit inspection of unit by the server

C_5 = Cost per unit inspection of unit by the server

C_6 = Cost per unit replacement of the unit

C_7 = Cost per unit replacement of the switch

C_8 = Cost per unit repair of the unit

C_9 = Cost per unit repair of the switch

And $A_0, B_0^I, B_0^R, T_0, I_0^u, I_0^s, P_0^u, P_0^s, O_0^u$, and O_0^s, are already defined.

6. Particular Case

The particular cases are calculated for Weibull density function with common shape parameter and different scale parameters. The probability density function for various random variables included in the model for Weibull case are as follows:

$z(t) = \lambda\eta t^{\eta-1}\exp(-\lambda t^{\eta})$, $f(t) = \alpha\eta t^{\eta-1}\exp(-\alpha t^{\eta})$, $r(t) = \mu\eta t^{\eta-1}\exp(-\mu t^{\eta})$, $h(t) = \gamma\eta t^{\eta-1}\exp(-\gamma t^{\eta})$,

$s(t) = \beta\eta t^{\eta-1}\exp(-\beta t^{\eta})$, $m(t) = \theta\eta t^{\eta-1}\exp(-\theta t^{\eta})$, $n(t) = \xi\eta t^{\eta-1}\exp(-\xi t^{\eta})$,

Where $t \geq 0$ and $\eta,\alpha,\lambda,\beta,\gamma,\mu,\theta,\xi > 0$

Changes made in various parameters of failure rate and repair rate -

Unit's repair rate α vary from 0.4 to 0.7.

Server's failure rate μ vary from o.3 to 0.002.

Server's repair rate β from 0.6 to 0.9.

Switch's repair rate γ from 0.5 to 0.7.

Switch's operating/failure probability p/q from 0.8/0.2 to 0.7/0.3.

Unit's inspection rate θ from 0.6 to 0.8.

Unit's repair/replacement probability after inspection a/b from 0.7/0.3 to 0.9/0.1.

Switch's inspection rate ξ from 0.7 to 0.9.

Switch's repair/replacement probability after inspection c/d from 0.6/0.4 to 0.8/0.2.

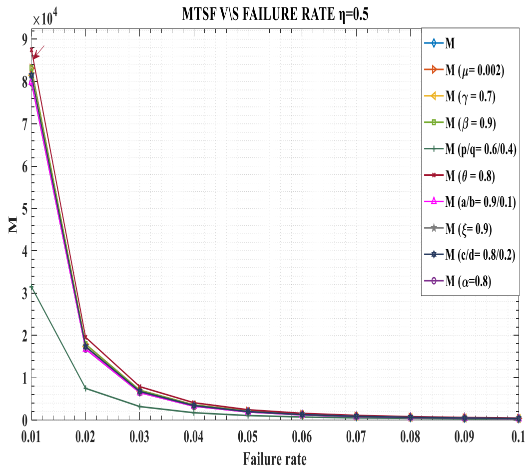

Figure 2(a): *MTSF w.r.t. failure rate when η=0.5*

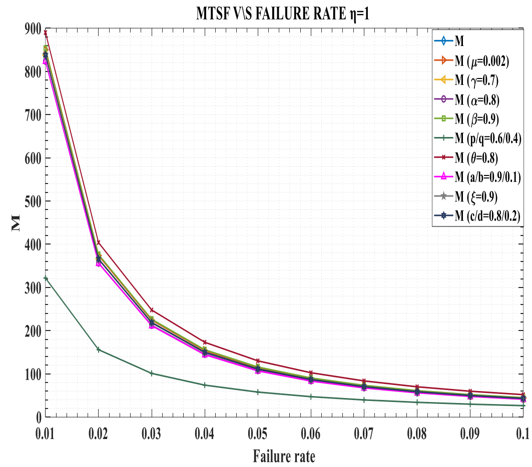

Figure 2(b): *MTSF w.r.t. failure rate when η=1*

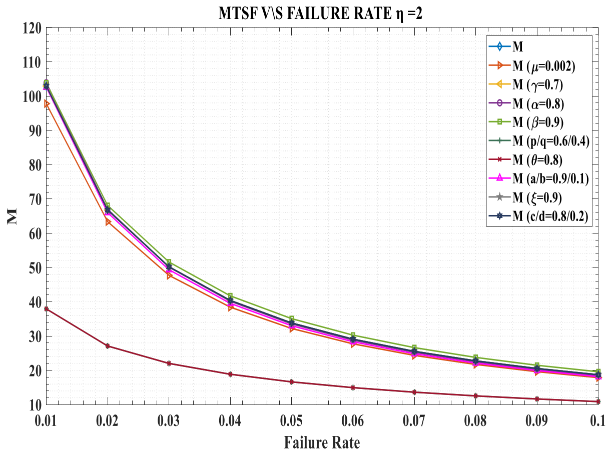

Figure 2(c): *MTSF w.r.t. failure rate when η=2*

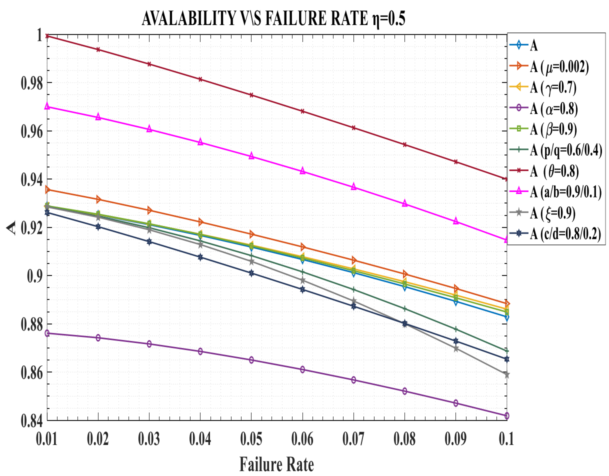

Figure 3(a): *Availability w.r.t. failure rate when η=0.5*

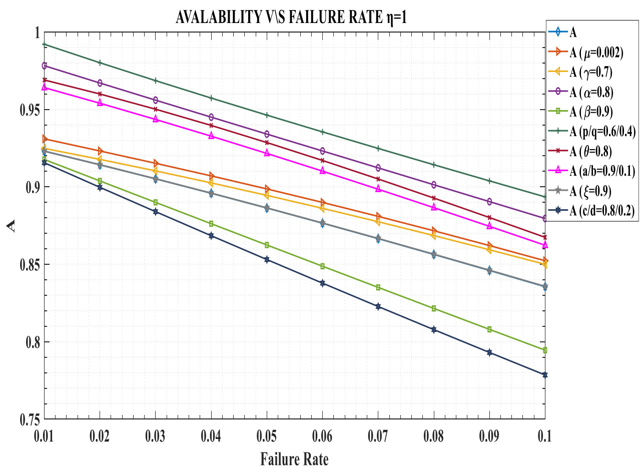

Figure 3(b): *Availability w.r.t. failure rate when η=1*

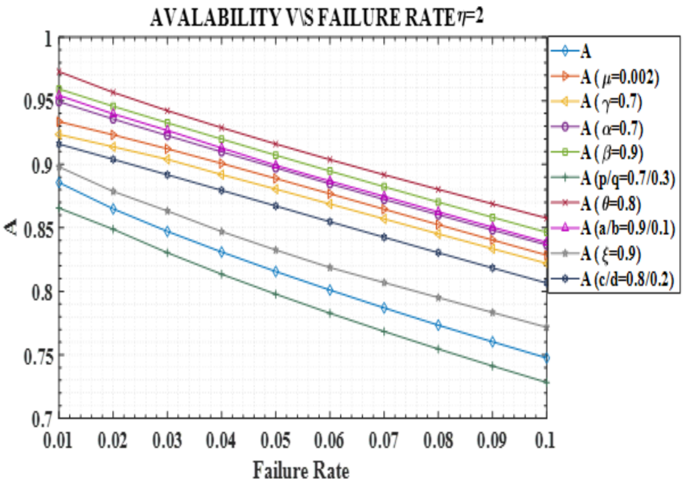

Figure 3(c): *Availability w.r.t. failure rate when η=2*

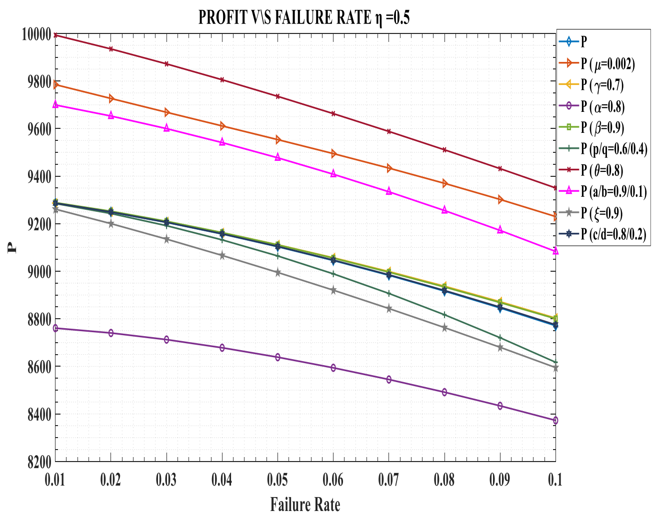

Figure 4(a): *Profit w.r.t. failure rate when η=0.5*

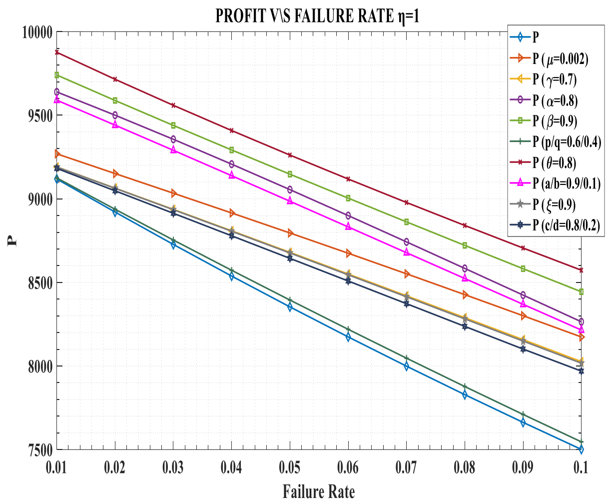

Figure 4(b): *Profit w.r.t. failure rate when η=1*

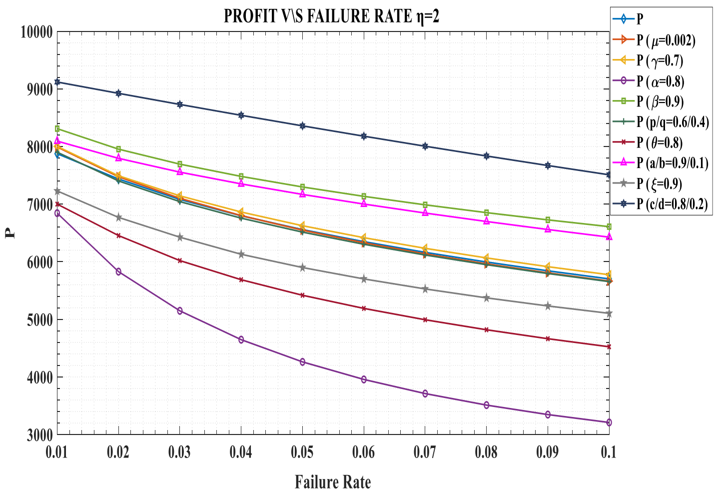

Figure 4(c): *Profit w.r.t. failure rate when η=2*

Mean time to failure (MTSF) is a crucial part in assessing system performance, especially in fields such as engineering and technology. In this context, Figures 2(a), 2(b), and 2(c) depict the behavior of MTSF for different values of failure rate and size parameters. A notable observation is that as the failure rate (λ) increases, the MTSF decreases. This trend is important because it implies that higher failure rates correspond to lower mean time to failure, which can affect the overall efficiency of the system. The failure time distribution is another important aspect affected by parameters such as magnitude (η). Figures 3(a), 3(b), and 3(c) depict similar behavior in terms of availability and failure rate. When the magnitude parameter is less than one, the output decays rapidly, indicating a decrease in performance and reliability over time. In contrast, when the magnitude parameter is greater than one, it suggests an increasing availability trend with higher inspection rates. Figures 4(a), 4(b), and 4(c) show the gain, which exhibits a decreasing trend as the shape parameter (η) increases. This implies that the gain reduces as the inspection rate increases, suggesting a saturation point beyond which further inspection does not significantly improve system performance. However, it is important to note that even though the gain reduces, inspection still has a positive impact on the overall system performance due to its high efficiency and availability.

7. Conclusion

Our research leads us to the conclusion that the evaluation of a malfunctioning device and the subsequent determination of whether to repair or replace it play a crucial role in influencing system performance and availability. While repair may incur greater costs and time commitments, our study highlights that replacing the defective component post-failure stands out as the most effective solution, particularly in terms of enhancing system availability and reliability within server environments. These principles hold significant relevance in critical and hard-to-reach systems, where the failure of a single module can trigger substantial operational disruptions. Illustrative examples include DSLAM networks, wind power plants, hydra production facilities, and automatic plastering machines, underscoring the widespread application of these concepts across diverse industrial settings.

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