ESTIMATION OF RELIABILITY ON SEQUENTIAL ORDER STATISTICS FROM (k, n) SYSTEM

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Abstract

The focus of this paper is to introduce a reliability model for differently structured independent sequential (k, n) systems. In such a system, the failure of any component possibly influences the other components such that their underlying failure rate is parametrically adjusted with respect to the number of preceding failures. The system works if and only if at least k out of the n components works. By considering the different models of sequential (k, n) system, we obtain the reliability assuming that the system failure time belongs to exponential/gamma distribution with location and scale parameters. These results are important because the distributions can model diverse time-to-failure behavior. As the result it is found that the reliability decreases with increase in time by shifting location and scale parameters. This indicates that the reliability for different models of sequential (k, n) system are as expected.

Keywords: Sequential (k, n) systems, Reliability function, Exponential/Gamma distribution, Location and Scale parameters.

1. Introduction

Components designed to carry out a specific task make up a system. The component failure times are commonly assumed to be independent and identically distributed in the (k, n) system. The remaining operational components are supposed to remain unaffected by any component failure in the system. However, this is practically not applicable. Any failure in one of the system's components will impact the system as a whole. This places additional strain on the remaining active components, leading to an increase in stress levels. This leads to a rise in failure rates, a fall in efficiency, or both. As a result, an alternative flexible model was developed by taking into account variations in the active component's lifelengths distribution. In this model, as and when each component fails, the remaining active components take on the stress and change their distribution. The extensible model, developed specifically for this reason, is termed as a sequential (k, n) system. Sequential order statistics are the results of the sequential (k, n) system's order statistics. In this case, we suppose that the failure rate for the surviving active components changes with each component failure. A (k, n) system is said to be a sequential (k, n) system wherein lifelengths distribution of the active components changes if any one of the components fails. The r^{th} sequential order statistics model the life length of a sequential $(n - r + 1, n)$ system.

Cramer and Kamps [7] derived the basic results pertaining to the sequential order statistics. Bain [2], Barlow [4], and Meeker et al. [15] provide the most useful statistical techniques in the areas of reliability and life testing models. Basu and Mawaziny [3] identified the minimum variance unbiased estimator of the system's reliability at a given mission time. Baratnia and Doostparast [5] describe the lifetime of engineering systems when component lifetimes are dependent. Pham [18] discussed the most likely estimates of reliability and the uniformly minimal variance unbiased estimator for k-out-of-n systems. These systems consist of n independent, identically distributed components with exponential lifetimes. Méndez-González et al. [16] used the inverse power law and the exponentiated Weibull model to examine the reliability of an electronic component. Using the exponentiated Weibull distribution, Chaturvedi and Pathak [6] were able to get the reliability function's Maximum likelihood estimator. Demiray and Kizilaslan [9] looked in k-out-of-n system stress-strength reliability using point and interval estimates where stress and strength variable follows the proportional hazard rate model. Kalaivani and Kannan [12] introduced the concept of mean time to system failure and reliability function for (k,n) system using weibull distribution. Shi et al. [20] with Burr XII components used both classical and Bayes approaches to investigate how well the m consecutive (k, n): F system worked. Alghamdi and Percy [1] looked at the equivalence and reliability factors of a series-parallel system following exponentiated Weibull distribution. According to Hong and Meeker [11], the system's reliability is determined by the components and system structure.

This article aims to determine the reliability function based on sequential order statistics from different models of sequential (k, n) systems, under the assumption that the failure time distribution follows an exponential/gamma location-scale family. The article is organized as follows: In Section 2, the distribution function, marginal and joint density function of different models of sequential (k, n) systems are given. Section 3 calculates the reliability function of (1, 3) and (2, 3) systems. . Section 4 calculates the reliability function of (1, 4) and (2, 4) systems. In Section 5, numerical illustrations are presented for analysis, and Section 6 gives the conclusion about the result.

2. Sequential Order Statistics

According to Kamps [13], the joint density function of the first $r, (1 \leq r \leq n)$ sequential order statistics $X_*^{(1)}, X_*^{(2)}, \dots, X_*^{(r)}$ based on absolutely continuous distribution functions $F_1, F_2, \dots, F_n$ with respective density functions $f_1, f_2, \dots, f_n$ is given by

$$f^*(x_1, x_2, \dots, x_r) = \frac{n!}{(n-r)!} \prod_{i=1}^{r} \left[\left\{ \frac{1 - F_i(x_i)}{1 - F_i(x_{i-1})} \right\}^{n-i} \frac{f_i(x_i)}{1 - F_i(x_{i-1})} \right],$$

$$-\infty = x_0 < x_1 < \dots < x_n < \infty.$$

Revathy and Chandrasekar [19] provide certain reliability metrics and equivariant parameter estimates based on sequential $(1, 3)$ and $(2, 3)$ systems. By introducing sequential $(1, 4)$ and $(2, 4)$ systems, Glory Prasanth and Venmani [10] computed the minimum risk equivariant estimator of location, scale, and location-scale families. In this section, we discuss about the distribution function, marginal and joint density function of system failure time for different models of sequential (k, n) system.

The density function of the random variable X with Gamma distribution (δ, τ, k) where δ is the location parameter, τ is the scale parameter and k is the shape parameter is defined as

$$f(x; \delta, \tau, k) = \frac{1}{(k-1)!\, \tau^k} (x - \delta)^{k-1} e^{-\frac{1}{\tau}(x-\delta)} \; ; \; \delta > 0, \tau > 0, k > 0, x > 0.$$

2.1. Model 1

Consider the sequential (1, 3) system which are absolutely continuous with the lifelength distributions F_1, F_2, F_3 having the respective density functions f_1, f_2, f_3. Let f_1 and f_2 be the density function of Gamma distribution $(\delta, \tau, 2)$ and f_3 the density function of Gamma distribution $(\delta, \tau, 1)$. The sequential order statistics have the failure times as $X_*^{(1)}, X_*^{(2)}$ and $X_*^{(3)}$.

Suppose $F_1(x) = F_2(x) = 1 - e^{-\frac{1}{\tau}(x-\delta)}\left[1 + \frac{x-\delta}{\tau}\right], \;\; \tau > 0, \; x > \delta, \; \delta \in R$

and $F_3(x) = 1 - e^{-\frac{1}{\tau}(x-\delta)}, \; \tau > 0, \; x > \delta, \; \delta \in R.$

Then $f_1(x) = f_2(x) = \frac{1}{\tau^2}(x-\delta)e^{-\frac{1}{\tau}(x-\delta)}, \;\; \tau > 0, \; x > \delta, \; \delta \in R$

and $f_3(x) = \frac{1}{\tau}e^{-\frac{1}{\tau}(x-\delta)}, \;\; \tau > 0, \; x > \delta, \; \delta \in R.$

Thus the joint probability density functions of $X_*^{(1)}, X_*^{(2)}$ and $X_*^{(3)}$ is

$$f^*(x_1, x_2, x_3) = 6\frac{1}{\tau^5}(x_1 - \delta)(x_2 - \delta)\left[1 + \frac{x_2 - \delta}{\tau}\right]e^{-\frac{1}{\tau}(x_1 + x_2 + x_3 - 3\delta)}, \quad \delta < x_1 < x_2 < x_3 < \infty, \; \delta \epsilon R, \; \tau > 0. \tag{1}$$

2.2. Model 2

Consider the sequential (2, 3) system which are absolutely continuous with the lifelength distributions F_1, F_2 having the respective density functions f_1, f_2. Let f_1 be the density function of Gamma distribution $(\delta, \tau, 2)$ and f_2 be the density function of Gamma distribution $(\delta, \tau, 1)$. Let $X_*^{(1)}$ and $X_*^{(2)}$ be the failure times which are called sequential order statistics.

Suppose $F_1(x) = 1 - e^{-\frac{1}{\tau}(x-\delta)}\left[1 + \frac{x-\delta}{\tau}\right], \;\; \tau > 0, \; x > \delta, \; \delta \in R$

and $F_2(x) = 1 - e^{-\frac{1}{\tau}(x-\delta)}, \; \tau > 0, \; x > \delta, \; \delta \in R.$

Then $f_1(x) = \frac{1}{\tau^2}(x-\delta)e^{-\frac{1}{\tau}(x-\delta)}, \;\; \tau > 0, \; x > \delta, \; \delta \in R$

and $f_2(x) = \frac{1}{\tau}e^{-\frac{1}{\tau}(x-\delta)}, \;\; \tau > 0, \; x > \delta, \; \delta \in R.$

Thus the joint probability density function of $X_*^{(1)}$ and $X_*^{(2)}$ is

$$f^*(x_1, x_2) = 6\frac{1}{\tau^3}(x_1 - \delta)\left[1 + \frac{x_1 - \delta}{\tau}\right]^2 e^{-\frac{1}{\tau}(x_1 + 2x_2 - 3\delta)}, \delta < x_1 < x_2 < \infty, \delta \epsilon R, \tau > 0. \tag{2}$$

2.3. Model 3

Consider the sequential (1, 4) system which are absolutely continuous with the lifelength distributions F_1, F_2, F_3, F_4 having the respective density functions f_1, f_2, f_3, f_4. Let f_1 & f_2 be the density function of Gamma distribution $(\delta, \tau, 2)$ and f_3 & f_4 be the density function of Gamma distribution $(\delta, \tau, 1)$.

Suppose $F_1(x) = F_2(x) = 1 - e^{-\frac{1}{\tau}(x-\delta)}\left[1 + \frac{x-\delta}{\tau}\right], \; \tau > 0, \; x > \delta, \; \delta \in R$

and $F_3(x) = F_4(x) = 1 - e^{-\frac{1}{\tau}(x-\delta)},\ \tau > 0,\ x > \delta,\ \delta \in R.$

Then $f_1(x) = f_2(x) = \frac{1}{\tau^2}(x-\delta)e^{-\frac{1}{\tau}(x-\delta)},\ \tau > 0,\ x > \delta,\ \delta \in R$

and $f_3(x) = f_4(x) = \frac{1}{\tau}e^{-\frac{1}{\tau}(x-\delta)},\ \tau > 0,\ x > \delta,\ \delta \in R.$

Thus the joint probability density function of $X_*^{(1)}, X_*^{(2)}, X_*^{(3)}$and $X_*^{(4)}$ is

$$f^*(x_1, x_2, x_3, x_4) = \frac{24}{\tau^6}(x_1 - \delta)(x_2 - \delta)\left[1 + \frac{x_2 - \delta}{\tau}\right]^2 e^{-\frac{1}{\tau}(x_1+x_2+x_3+x_4-4\delta)},$$

$$\delta < x_1 < x_2 < x_3 < x_4 < \infty,\ \delta \epsilon R,\ \tau > 0. \tag{3}$$

2.4. Model 4

Consider the sequential (2, 4) system which are absolutely continuous with the lifelength distributions F_1 , F_2 , F_3 having the respective density functions f_1 , f_2 , f_3 . Let f_1 be the density function of Gamma distribution $(\delta, \tau, 2)$ and f_2 & f_3 be the density function of Gamma distribution $(\delta, \tau, 1)$. Let $X_*^{(1)}, X_*^{(2)}$and $X_*^{(3)}$ be the failure times which are called sequential order statistics.

Suppose $F_1(x) = 1 - e^{-\frac{1}{\tau}(x-\delta)}\left[1 + \frac{x-\delta}{\tau}\right],\ \tau > 0,\ x > \delta,\ \delta \in R$

and $F_2(x) = F_3(x) = 1 - e^{-\frac{1}{\tau}(x-\delta)},\ \tau > 0,\ x > \delta,\ \delta \in R.$

Then $f_1(x) = \frac{1}{\tau^2}(x-\delta)e^{-\frac{1}{\tau}(x-\delta)},\ \tau > 0,\ x > \delta,\ \delta \in R$

and $f_2(x) = f_3(x) = \frac{1}{\tau}e^{-\frac{1}{\tau}(x-\delta)},\ \tau > 0,\ x > \delta,\ \delta \in R.$

Thus the joint probability density function of $X_*^{(1)}, X_*^{(2)}$and $X_*^{(3)}$ is

$$f^*(x_1, x_2, x_3) = 24\frac{1}{\tau^4}(x_1 - \delta)\left[1 + \frac{x_1 - \delta}{\tau}\right]^3 e^{-\frac{1}{\tau}(x_1+x_2+2x_3-4\delta)},$$

$$\delta < x_1 < x_2 < x_3 < \infty,\ \delta \epsilon R,\ \tau > 0. \tag{4}$$

3. Reliability Measure of (1, 3) and (2, 3) systems

In this section, using the joint density function given in (1) & (2), the reliability function for different models of sequential (1,3) and (2,3) systems are obtained.

The Reliability function for (1, 3) system is

$$R(t) = \int\int\int f^*(x_1, x_2, x_3)\, dx_3 dx_2\, dx_1\ ,\ t < x_1, x_2, x_3 < \infty$$

$$= \int\int\int 6\frac{1}{\tau^5}(x_1 - \delta)(x_2 - \delta)\left[1 + \frac{x_2 - \delta}{\tau}\right] e^{-\frac{1}{\tau}(x_1+x_2+x_3-3\delta)}\, dx_3 dx_2\, dx_1\ ,\ t < x_1, x_2, x_3 < \infty$$

$$= \frac{6}{\tau^3}(t-\delta)^3 e^{-\frac{3}{\tau}(t-\delta)} + \frac{24}{\tau^2}(t-\delta)^2 e^{-\frac{3}{\tau}(t-\delta)} + \frac{36}{\tau}(t-\delta)e^{-\frac{3}{\tau}(t-\delta)} + 18e^{-\frac{3}{\tau}(t-\delta)},$$

$$t < x_1, x_2, x_3 < \infty, \delta > 0, \tau > 0.$$

The Reliability function for (2, 3) system is

$$R(t) = \int\int f^*(x_1, x_2) dx_2 dx_1 , t < x_1, x_2, < \infty$$

$$= \int\int 6\frac{1}{\tau^3}(x_1 - \delta)\left[1 + \frac{x_1 - \delta}{\tau}\right]^2 e^{-\frac{1}{\tau}(x_1+2x_2-3\delta)} dx_2 dx_1 , t < x_1, x_2, < \infty$$

$$= \frac{3}{\tau^3}(t-\delta)^3 e^{-\frac{3}{\tau}(t-\delta)} + \frac{15}{\tau^2}(t-\delta)^2 e^{-\frac{3}{\tau}(t-\delta)} + \frac{24}{\tau}(t-\delta) e^{-\frac{3}{\tau}(t-\delta)} + 24 e^{-\frac{3}{\tau}(t-\delta)},$$

$$t < x_1, x_2 < \infty, \delta > 0, \tau > 0.$$

4. Reliability Measure of (1, 4) and (2, 4) systems

In this section, using the joint density function given in (3) & (4), the reliability function for different models of sequential (1, 4) and (2, 4) systems are obtained.

The Reliability function for (1, 4) system is

$$R(t) = \int\int\int\int f^*(x_1, x_2, x_3, x_4) dx_4 dx_3 dx_2 dx_1 , t < x_1, x_2, x_3, x_4 < \infty$$

$$= \int\int\int\int \frac{24}{\tau^6}(x_1 - \delta)(x_2 - \delta)\left[1 + \frac{x_2 - \delta}{\tau}\right]^2 e^{-\frac{1}{\tau}(x_1+x_2+x_3+x_4-4\delta)} dx_4 dx_3 dx_2 dx_1,$$

$$t < x_1, x_2, x_3, x_4 < \infty$$

$$= \frac{24}{\tau^4}(t-\delta)^4 e^{-\frac{4}{\tau}(t-\delta)} + \frac{72}{\tau^3}(t-\delta)^3 e^{-\frac{4}{\tau}(t-\delta)} + \frac{312}{\tau^2}(t-\delta)^2 e^{-\frac{4}{\tau}(t-\delta)} + \frac{528}{\tau}(t-\delta) e^{-\frac{4}{\tau}(t-\delta)}$$

$$+ 264 e^{-\frac{4}{\tau}(t-\delta)}, \quad t < x_1, x_2, x_3, x_4 < \infty, \delta > 0, \tau > 0.$$

The Reliability function for (2, 4) system is

$$R(t) = \int\int\int f^*(x_1, x_2, x_3)\, dx_3 dx_2\, dx_1 ,\ t < x_1, x_2, x_3 < \infty$$

$$R(t) = \int\int\int 24\frac{1}{\tau^4}(x_1 - \delta)\left[1 + \frac{x_1 - \delta}{\tau}\right]^3 e^{-\frac{1}{\tau}(x_1+x_2+2x_3-4\delta)} dx_3 dx_2\, dx_1 ,\ t < x_1, x_2, x_3 < \infty$$

$$= \frac{12}{\tau^4}(t-\delta)^4 e^{-\frac{4}{\tau}(t-\delta)} + \frac{84}{\tau^3}(t-\delta)^3 e^{-\frac{4}{\tau}(t-\delta)} + \frac{288}{\tau^2}(t-\delta)^2 e^{-\frac{4}{\tau}(t-\delta)} + \frac{588}{\tau}(t-\delta) e^{-\frac{4}{\tau}(t-\delta)}$$

$$+ 588 e^{-\frac{4}{\tau}(t-\delta)}, \quad t < x_1, x_2, x_3 < \infty, \delta > 0, \tau > 0.$$

5. Numerical illustration

In this section, numerical illustration is presented for sequential (k, n) systems. The reliability R(t) for different time (t) with various location parameter and scale parameter when the failure time of the system follows exponential/gamma distribution are calculated. The reliability for different time by shifting location and scale parameters for different models of sequential (k, n) systems are presented in Table 1,2,3,4,5,6,7 and 8.

Table 1: *Reliability for (1, 3) system versus time for $\delta = 0.60$ and $\tau = 0.25, 0.50, 0.75, 1$*

S. No	δ=0.60 , τ=0.25		δ=0.60 , τ=0.5		δ=0.60 , τ=0.75		δ=0.60 , τ=1	
	t	R(t)	t	R(t)	t	R(t)	t	R(t)
1	0.89	0.97948	1.47	0.99987	1.91	0.98624	2.34	0.99987
2	0.91	0.77831	1.49	0.92062	1.93	0.93340	2.36	0.95948
3	0.93	0.61688	1.51	0.84726	1.95	0.88323	2.38	0.92062
4	0.95	0.48776	1.53	0.77941	1.97	0.83558	2.4	0.88323
5	0.97	0.38479	1.55	0.71669	1.99	0.79035	2.42	0.84726
6	0.99	0.30290	1.57	0.65873	2.01	0.74743	2.44	0.81267
7	1.01	0.23795	1.59	0.60522	2.03	0.70670	2.46	0.77941
8	1.03	0.18656	1.61	0.55583	2.05	0.66808	2.48	0.74743
9	1.05	0.14600	1.63	0.51027	2.07	0.63144	2.5	0.71669
10	1.07	0.11405	1.65	0.46827	2.09	0.59671	2.52	0.68713
11	1.09	0.08894	1.67	0.42956	2.11	0.56379	2.54	0.65873
12	1.11	0.06925	1.69	0.39391	2.13	0.53259	2.56	0.63144
13	1.13	0.05384	1.71	0.36109	2.15	0.50303	2.58	0.60522
14	1.15	0.04179	1.73	0.33088	2.17	0.47503	2.6	0.58003
15	1.17	0.03239	1.75	0.30310	2.19	0.44852	2.62	0.55583
16	1.19	0.02508	1.77	0.27756	2.21	0.42341	2.64	0.53259
17	1.21	0.01939	1.79	0.25409	2.23	0.39965	2.66	0.51027
18	1.23	0.01497	1.81	0.23252	2.25	0.37716	2.68	0.48884
19	1.25	0.01154	1.83	0.21272	2.27	0.35588	2.7	0.46827
20	1.27	0.00889	1.85	0.19455	2.29	0.33575	2.72	0.44852

Table 2: *Reliability for (1, 3) system versus time for $\tau = 0.65$ and $\delta = 0.2, 0.4, 0.6, 0.8$*

S. No	δ=0.2 , τ=0.65		δ=0.4 , τ=0.65		δ=0.6 , τ=0.65		δ=0.8 , τ=0.65	
	t	R(t)	t	R(t)	t	R(t)	t	R(t)
1	1.34	0.97174	1.54	0.97174	1.74	0.97174	2.15	0.98387
2	1.36	0.91186	1.56	0.91186	1.76	0.91186	2.17	0.93498
3	1.38	0.85544	1.58	0.85544	1.78	0.85544	2.19	0.88813
4	1.40	0.80230	1.6	0.80230	1.8	0.80230	2.21	0.84328
5	1.42	0.75227	1.62	0.75227	1.82	0.75227	2.23	0.80037
6	1.44	0.70518	1.64	0.70518	1.84	0.70518	2.25	0.75933
7	1.46	0.66088	1.66	0.66088	1.86	0.66088	2.27	0.72011
8	1.48	0.61921	1.68	0.61921	1.88	0.61921	2.29	0.68265
9	1.50	0.58003	1.7	0.58003	1.9	0.58003	2.31	0.64690
10	1.52	0.54320	1.72	0.54320	1.92	0.54320	2.33	0.61278
11	1.54	0.50859	1.74	0.50859	1.94	0.50859	2.35	0.58026
12	1.56	0.47608	1.76	0.47608	1.96	0.47608	2.37	0.54926
13	1.58	0.44555	1.78	0.44555	1.98	0.44555	2.39	0.51973
14	1.60	0.41689	1.80	0.41689	2.00	0.41689	2.41	0.49162
15	1.62	0.38998	1.82	0.38998	2.02	0.38998	2.43	0.46487
16	1.64	0.36474	1.84	0.36474	2.04	0.36474	2.45	0.43943
17	1.66	0.34106	1.86	0.34106	2.06	0.34106	2.47	0.41525
18	1.68	0.31885	1.88	0.31885	2.08	0.31885	2.49	0.39227
19	1.70	0.29802	1.9	0.29802	2.1	0.29802	2.51	0.37044
20	1.72	0.27850	1.92	0.27850	2.12	0.27850	2.53	0.34972

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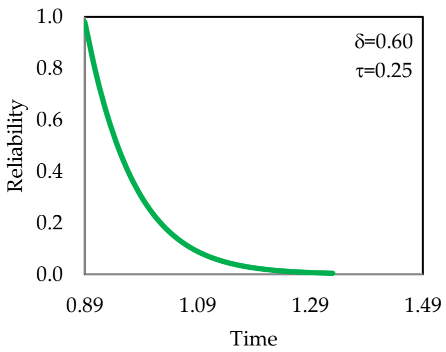

Figure 1: *Reliability versus time with* δ *= 0.60,* τ *= 0.25 for (1, 3) system.*

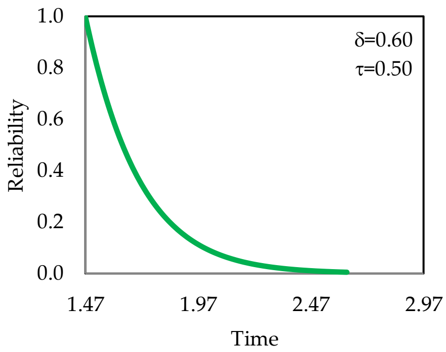

Figure 2: *Reliability versus time with* δ *= 0.60,* τ *= 0.50 for (1, 3) system.*

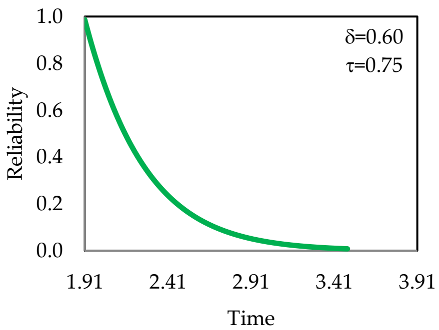

Figure 3: *Reliability versus time with* δ *= 0.60,* τ *= 0.75 for (1, 3) system.*

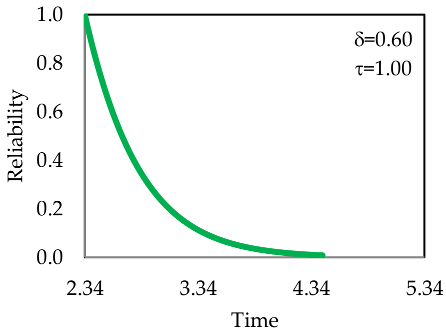

Figure 4: *Reliability versus time with* δ *= 0.60,* τ *= 1.00 for (1, 3) system.*

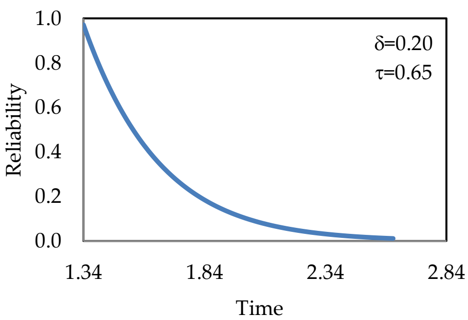

Figure 5: *Reliability versus time with* δ *= 0.20,* τ *= 0.65 for (1, 3) system.*

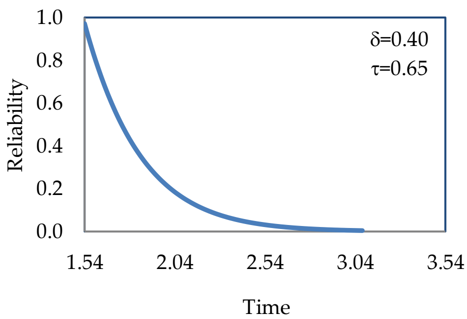

Figure 6: *Reliability versus time with* δ *= 0.40,* τ *= 0.65 for (1, 3) system.*

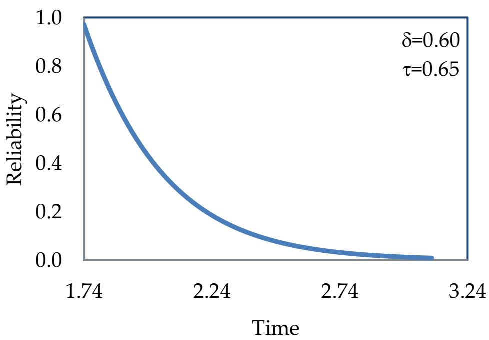

Figure 7: *Reliability versus time with* δ *= 0.60,* τ *= 0.65 for (1, 3) system.*

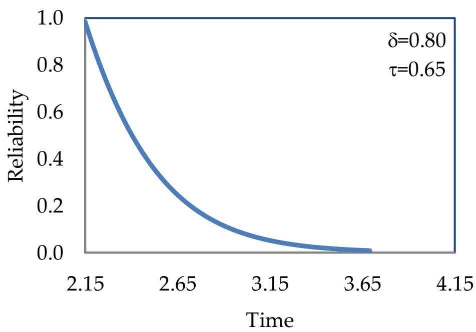

Figure 8: *Reliability versus time with* δ *= 0.80 ,* τ *= 0.65 for (1, 3) system*

From Table 1 & 2, it is observed that the sequential (1, 3) system's reliability decreases with increasing time and is also clear that time increases along with location and scale parameters.

Table 3: *Reliability for (2, 3) system versus time for $\delta = 0.60$ and $\tau = 0.25, 0.50, 0.75, 1$*

S. No	δ=0.60 , τ=0.25		δ=0.60 , τ=0.5		δ=0.60 , τ=0.75		δ=0.60 , τ=1	
	t	R(t)	t	R(t)	t	R(t)	t	R(t)
1	0.99	1.00000	1.51	1.00000	1.77	1.00000	2.34	0.99987
2	1.01	0.85354	1.53	0.93633	1.79	0.95786	2.36	0.95948
3	1.03	0.71726	1.55	0.86702	1.81	0.90425	2.38	0.92062
4	1.05	0.60204	1.57	0.80246	1.83	0.85354	2.4	0.88323
5	1.07	0.50476	1.59	0.74235	1.85	0.80556	2.42	0.84726
6	1.09	0.42272	1.61	0.68642	1.87	0.76018	2.44	0.81267
7	1.11	0.35363	1.63	0.63441	1.89	0.71726	2.46	0.77941
8	1.13	0.29551	1.65	0.58607	1.91	0.67668	2.48	0.74743
9	1.15	0.24669	1.67	0.54116	1.93	0.63831	2.5	0.71669
10	1.17	0.20573	1.69	0.49948	1.95	0.60204	2.52	0.68713
11	1.19	0.17140	1.71	0.46080	1.97	0.56776	2.54	0.65873
12	1.21	0.14266	1.73	0.42493	1.99	0.53537	2.56	0.63144
13	1.23	0.11863	1.75	0.39169	2.01	0.50476	2.58	0.60522
14	1.25	0.09855	1.77	0.36090	2.03	0.47584	2.6	0.58003
15	1.27	0.08180	1.79	0.33239	2.05	0.44852	2.62	0.55583
16	1.29	0.06784	1.81	0.30601	2.07	0.42272	2.64	0.53259
17	1.31	0.05621	1.83	0.28161	2.09	0.39835	2.66	0.51027
18	1.33	0.04654	1.85	0.25905	2.11	0.37535	2.68	0.48884
19	1.35	0.03850	1.87	0.23821	2.13	0.35363	2.70	0.46827
20	1.37	0.03183	1.89	0.21896	2.15	0.33313	2.72	0.44852

Table 4: *Reliability for (2, 3) system versus time for $\tau = 0.65$ and $\delta = 0.2, 0.4, 0.6, 0.8$*

S. No	δ=0.2 , τ=0.65		δ=0.4 , τ=0.65		δ=0.6 , τ=0.65		δ=0.8 , τ=0.65	
	t	R(t)	t	R(t)	t	R(t)	t	R(t)
1	1.22	0.99454	1.42	0.99454	1.72	1.00000	1.88	0.99508
2	1.24	0.93069	1.44	0.93069	1.74	0.97174	1.90	0.93731
3	1.26	0.87078	1.46	0.87078	1.76	0.91186	1.92	0.88257
4	1.28	0.81458	1.48	0.81458	1.78	0.85544	1.94	0.83074
5	1.30	0.76188	1.50	0.76188	1.80	0.80230	1.96	0.78167
6	1.32	0.71246	1.52	0.71246	1.82	0.75227	1.98	0.73525
7	1.34	0.66614	1.54	0.66614	1.84	0.70518	2.00	0.69135
8	1.36	0.62272	1.56	0.62272	1.86	0.66088	2.02	0.64985
9	1.38	0.58203	1.58	0.58203	1.88	0.61921	2.04	0.61064
10	1.40	0.54391	1.60	0.54391	1.90	0.58003	2.06	0.57361
11	1.42	0.50820	1.62	0.50820	1.92	0.54320	2.08	0.53865
12	1.44	0.47476	1.64	0.47476	1.94	0.50859	2.10	0.50567
13	1.46	0.44344	1.66	0.44344	1.96	0.47608	2.12	0.47455
14	1.48	0.41413	1.68	0.41413	1.98	0.44555	2.14	0.44522
15	1.50	0.38669	1.70	0.38669	2.00	0.41689	2.15	0.43119
16	1.52	0.36101	1.72	0.36101	2.02	0.38998	2.17	0.40436
17	1.54	0.33698	1.74	0.33698	2.04	0.36474	2.19	0.37908
18	1.56	0.31450	1.76	0.31450	2.06	0.34106	2.21	0.35528
19	1.58	0.29348	1.78	0.29348	2.08	0.31885	2.23	0.33289
20	1.60	0.27381	1.80	0.27381	2.10	0.29802	2.25	0.31182

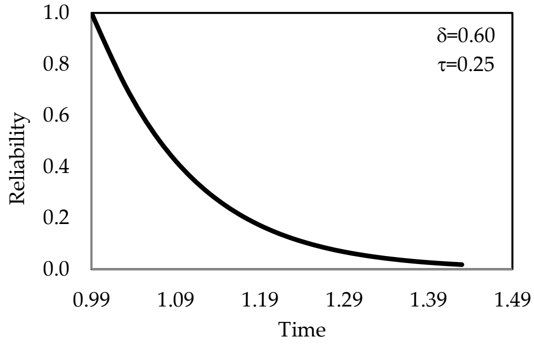

Figure 9: *Reliability versus time with δ = 0.60, τ = 0.25 for (2, 3) system.*

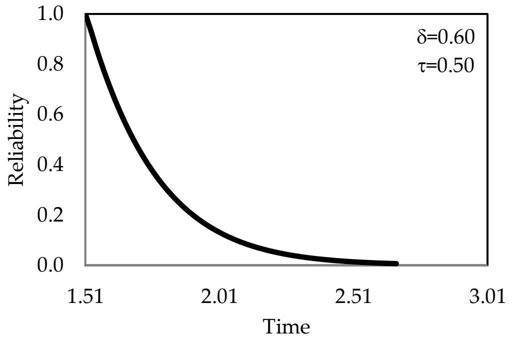

Figure 10: *Reliability versus time with δ = 0.60, τ = 0.50 for (2, 3) system.*

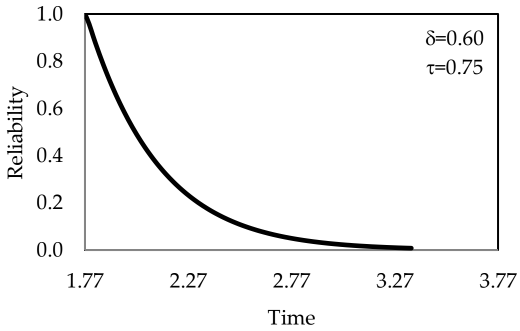

Figure 11: *Reliability versus time with δ = 0.60, τ = 0.75 for (2, 3) system.*

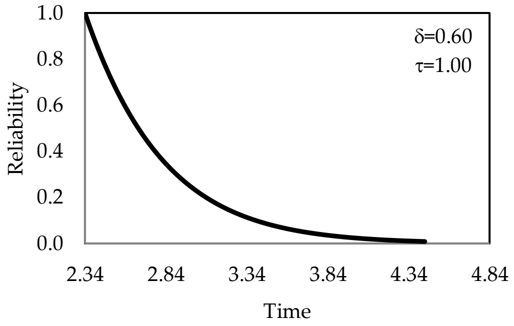

Figure 12: *Reliability versus time with δ = 0.60, τ = 1 for (2, 3) system.*

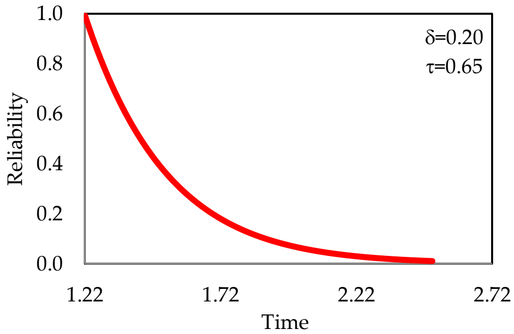

Figure 13: *Reliability versus time with δ = 0.2, τ = 0.65 for (2, 3) system.*

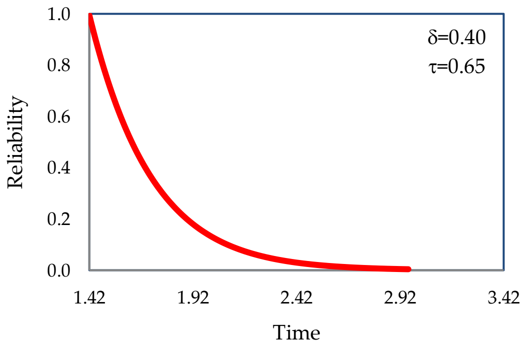

Figure 14: *Reliability versus time with δ = 0.40, τ = 0.65 for (2, 3) system.*

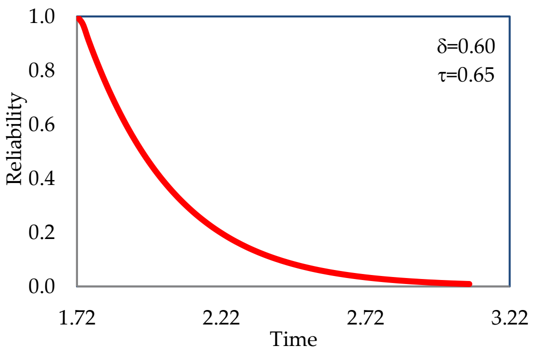

Figure 15: *Reliability versus time with δ = 0.6, τ = 0.65 for (2, 3) system.*

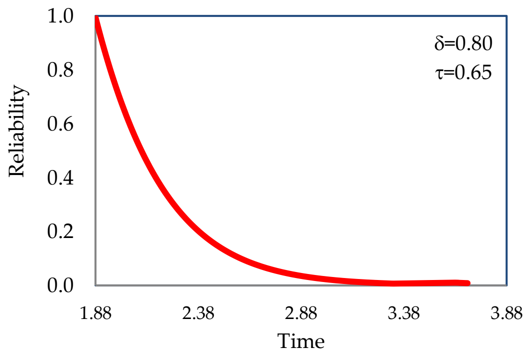

Figure 16: *Reliability versus time with δ = 0.8, τ = 0.65 for (2, 3) system.*

From Table 3 & 4, it is observed that the sequential (2, 3) system's reliability declines with increasing time and is also clear that time increases along with location and scale parameters.

Table 5: *Reliability for (1, 4) system versus time for δ=0.60 and τ=0.25, 0.50, 0.75, 1*

S. No	δ=0.60 , τ=0.25		δ=0.60 , τ=0.5		δ=0.60 , τ=0.75		δ=0.60 , τ=1	
	t	R(t)	t	R(t)	t	R(t)	t	R(t)
1	1.12	0.92791	1.64	0.92791	2.14	1.00000	2.66	0.98621
2	1.14	0.72668	1.66	0.82127	2.16	0.92791	2.68	0.92791
3	1.16	0.56846	1.68	0.72668	2.18	0.85541	2.70	0.87299
4	1.18	0.44422	1.70	0.64281	2.20	0.78847	2.72	0.82127
5	1.20	0.34677	1.72	0.56846	2.22	0.72668	2.74	0.77256
6	1.22	0.27042	1.74	0.50258	2.24	0.66965	2.76	0.72668
7	1.24	0.21068	1.76	0.44422	2.26	0.61702	2.78	0.68348
8	1.26	0.16397	1.78	0.39253	2.28	0.56846	2.80	0.64281
9	1.28	0.12751	1.80	0.34677	2.30	0.52366	2.82	0.60451
10	1.30	0.09906	1.82	0.30626	2.32	0.48233	2.84	0.56846
11	1.32	0.07689	1.84	0.27042	2.34	0.44422	2.86	0.53452
12	1.34	0.05963	1.86	0.23871	2.36	0.40906	2.88	0.50258
13	1.36	0.04621	1.88	0.21068	2.38	0.37665	2.90	0.47251
14	1.38	0.03577	1.90	0.18589	2.40	0.34677	2.92	0.44422
15	1.40	0.02767	1.92	0.16397	2.42	0.31922	2.94	0.41759
16	1.42	0.02139	1.94	0.14461	2.44	0.29382	2.96	0.39253
17	1.44	0.01652	1.96	0.12751	2.46	0.27042	2.98	0.36895
18	1.46	0.01275	1.98	0.11240	2.48	0.24885	3.00	0.34677
19	1.48	0.00984	2.00	0.09906	2.50	0.22898	3.02	0.32589
20	1.50	0.00758	2.02	0.08728	2.52	0.21068	3.04	0.30626

Table 6: *Reliability for (1, 4) system with time for τ = 0.65 and δ = 0.2, 0.4, 0.6, 0.8*

S. No	δ=0.2 , τ=0.65		δ=0.4 , τ=0.65		δ=0.6 , τ=0.65		δ=0.8 , τ=0.65	
	t	R(t)	t	R(t)	t	R(t)	t	R(t)
1	1.54	0.98160	1.74	0.98160	1.94	0.98160	2.14	0.98160
2	1.56	0.89373	1.76	0.89373	1.96	0.89373	2.16	0.89373
3	1.58	0.81358	1.78	0.81358	1.98	0.81358	2.18	0.81358
4	1.60	0.74050	1.80	0.74050	2.00	0.74050	2.20	0.74050
5	1.62	0.67388	1.82	0.67388	2.02	0.67388	2.22	0.67388
6	1.64	0.61315	1.84	0.61315	2.04	0.61315	2.24	0.61315
7	1.66	0.55780	1.86	0.55780	2.06	0.55780	2.26	0.55780
8	1.68	0.50737	1.88	0.50737	2.08	0.50737	2.28	0.50737
9	1.70	0.46143	1.90	0.46143	2.10	0.46143	2.30	0.46143
10	1.72	0.41958	1.92	0.41958	2.12	0.41958	2.32	0.41958
11	1.74	0.38147	1.94	0.38147	2.14	0.38147	2.34	0.38147
12	1.76	0.34677	1.96	0.34677	2.16	0.34677	2.36	0.34677
13	1.78	0.31517	1.98	0.31517	2.18	0.31517	2.38	0.31517
14	1.80	0.28642	2.00	0.28642	2.20	0.28642	2.40	0.28642
15	1.82	0.26025	2.02	0.26025	2.22	0.26025	2.42	0.26025
16	1.84	0.23643	2.04	0.23643	2.24	0.23643	2.44	0.23643
17	1.86	0.21477	2.06	0.21477	2.26	0.21477	2.46	0.21477
18	1.88	0.19506	2.08	0.19506	2.28	0.19506	2.48	0.19506
19	1.90	0.17714	2.10	0.17714	2.30	0.17714	2.50	0.17714
20	1.92	0.16084	2.12	0.16084	2.32	0.16084	2.52	0.16084

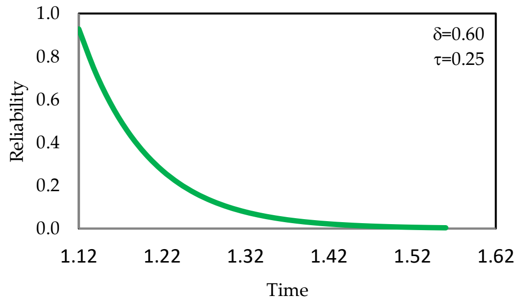

Figure 17: *Reliability versus time with $\delta = 0.60, \tau = 0.25$ for (1, 4) system.*

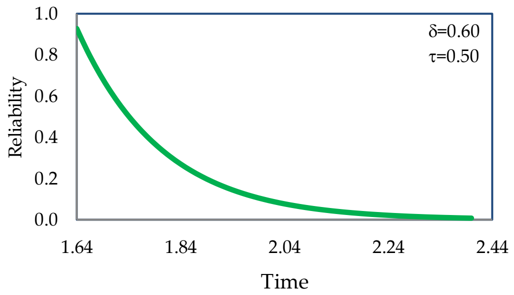

Figure 18: *Reliability versus time with $\delta = 0.60, \tau = 0.50$ for (1, 4) system.*

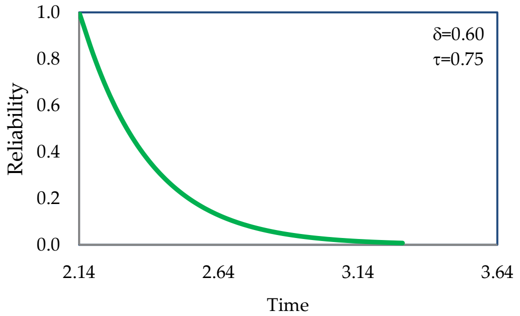

Figure 19: *Reliability versus time with $\delta = 0.60, \tau = 0.75$ for (1, 4) system.*

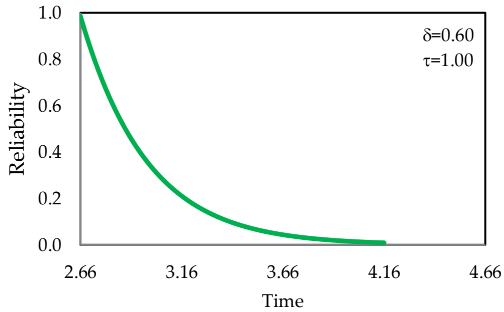

Figure 20: *Reliability versus time with $\delta = 0.60, \tau = 1.00$ for (1, 4) system.*

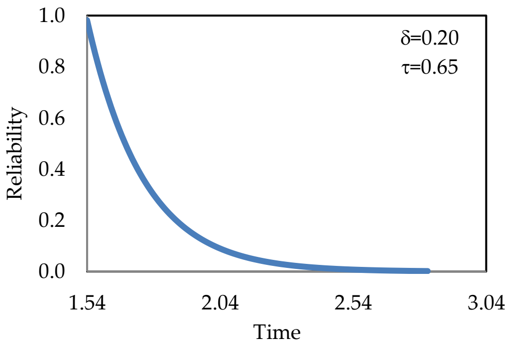

Figure 21: *Reliability versus time with $\delta = 0.20, \tau = 0.65$ for (1, 4) system.*

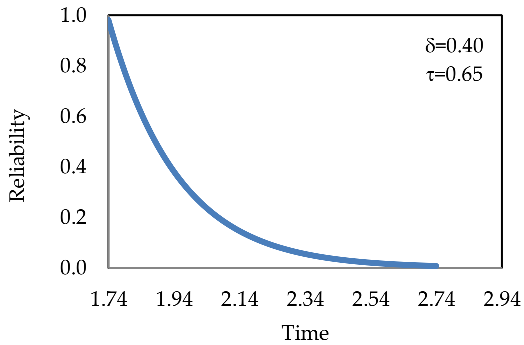

Figure 22: *Reliability versus time with $\delta = 0.40, \tau = 0.65$ for (1, 4) system.*

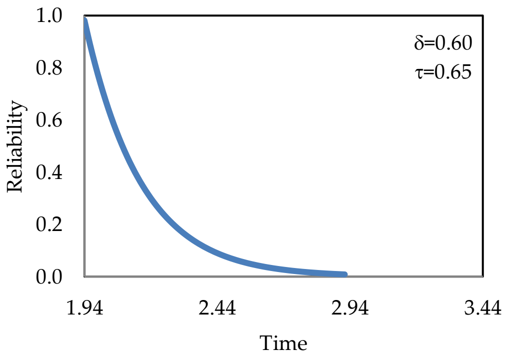

Figure 23: *Reliability versus time with $\delta = 0.60, \tau = 0.65$ for (1, 4) system.*

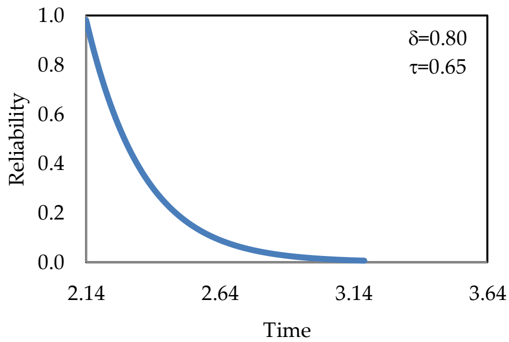

Figure 24: *Reliability versus time with $\delta = 0.8, \tau = 0.65$ for (1, 4) system.*

From Table 5 & 6, it can be observed that for sequential (1, 4) system's reliability declines with increasing time and it is also clear that time increases along with location and scale parameters.

Table 7: *Reliability for (2, 4) system with time for δ=0.60 and τ=0.25, 0.50, 0.75, 1*

S. No	δ=0.60 , τ=0.25		δ=0.60 , τ=0.5		δ=0.60 , τ=0.75		δ=0.60 , τ=1	
	t	R(t)	t	R(t)	t	R(t)	t	R(t)
1	1.12	0.9835	1.64	0.9835	2.16	0.98353	2.68	0.9835
2	1.14	0.7623	1.66	0.8660	2.18	0.90352	2.70	0.9229
3	1.16	0.5904	1.68	0.7623	2.20	0.82994	2.72	0.8660
4	1.18	0.4569	1.70	0.6709	2.22	0.76230	2.74	0.8125
5	1.20	0.3534	1.72	0.5904	2.24	0.70011	2.76	0.7623
6	1.22	0.2731	1.74	0.5194	2.26	0.64294	2.78	0.7152
7	1.24	0.2109	1.76	0.4569	2.28	0.59039	2.80	0.6709
8	1.26	0.1628	1.78	0.4019	2.30	0.54210	2.82	0.6294
9	1.28	0.1255	1.80	0.3534	2.32	0.49771	2.84	0.5904
10	1.30	0.0967	1.82	0.3107	2.34	0.45692	2.86	0.5538
11	1.32	0.0745	1.84	0.2731	2.36	0.41944	2.88	0.5194
12	1.34	0.0573	1.86	0.2400	2.38	0.38501	2.90	0.4872
13	1.36	0.0441	1.88	0.2109	2.40	0.35337	2.92	0.4569
14	1.38	0.0339	1.90	0.1853	2.42	0.32431	2.94	0.4285
15	1.40	0.0260	1.92	0.1628	2.44	0.29761	2.96	0.4019
16	1.42	0.0200	1.94	0.1429	2.46	0.27309	2.98	0.3768
17	1.44	0.0153	1.96	0.1255	2.48	0.25057	3.00	0.3534
18	1.46	0.0118	1.98	0.1102	2.50	0.22989	3.02	0.3313
19	1.48	0.0090	2.00	0.0967	2.52	0.21090	3.04	0.3107
20	1.50	0.0069	2.02	0.0849	2.54	0.19346	3.06	0.2913

Table 8: *Reliability for (2, 4) system with time for τ= 0.65 and δ= 0.2, 0.4, 0.6, 0.8*

S. No	δ=0.2 , τ=0.65		δ=0.4 , τ=0.65		δ=0.6 , τ=0.65		δ=0.8 , τ=0.65	
	t	R(t)	t	R(t)	t	R(t)	t	R(t)
1	1.56	0.99910	1.75	0.99321	1.95	0.99321	2.15	0.99321
2	1.58	0.90750	1.77	0.90057	1.97	0.90057	2.17	0.90057
3	1.60	0.82423	1.79	0.81649	1.99	0.81649	2.19	0.81649
4	1.62	0.74852	1.81	0.74017	2.01	0.74017	2.21	0.74017
5	1.64	0.67971	1.83	0.67092	2.03	0.67092	2.23	0.67092
6	1.66	0.61716	1.85	0.60808	2.05	0.60808	2.25	0.60808
7	1.68	0.56032	1.87	0.55107	2.07	0.55107	2.27	0.55107
8	1.70	0.50866	1.89	0.49935	2.09	0.49935	2.29	0.49935
9	1.72	0.46173	1.91	0.45243	2.11	0.45243	2.31	0.45243
10	1.74	0.41908	1.93	0.40988	2.13	0.40988	2.33	0.40988
11	1.76	0.38033	1.95	0.37129	2.15	0.37129	2.35	0.37129
12	1.78	0.34514	1.97	0.33630	2.17	0.33630	2.37	0.33630
13	1.80	0.31317	1.99	0.30458	2.19	0.30458	2.39	0.30458
14	1.82	0.28414	2.01	0.27581	2.21	0.27581	2.41	0.27581
15	1.84	0.25777	2.03	0.24974	2.23	0.24974	2.43	0.24974
16	1.86	0.23382	2.05	0.22611	2.25	0.22611	2.45	0.22611
17	1.88	0.21208	2.07	0.20469	2.27	0.20469	2.47	0.20469
18	1.90	0.19234	2.09	0.18529	2.29	0.18529	2.49	0.18529
19	1.92	0.17443	2.11	0.16770	2.31	0.16770	2.51	0.16770
20	1.94	0.15816	2.13	0.15177	2.33	0.15177	2.53	0.15177

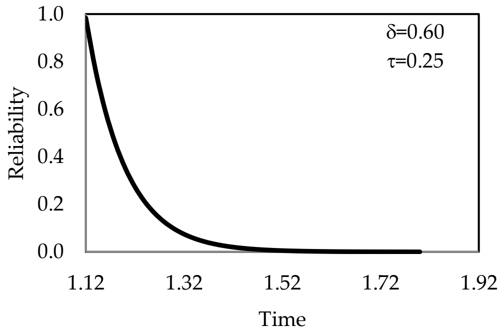

Figure 25: *Reliability versus time with $\delta = 0.60$, $\tau = 0.25$ for (2, 4) system.*

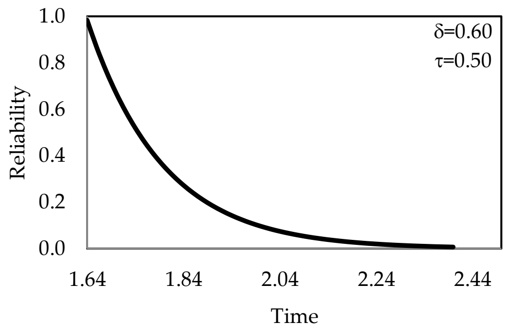

Figure 26:*Reliability versus time with $\delta = 0.60$, $\tau = 0.50$ for (2, 4) system.*

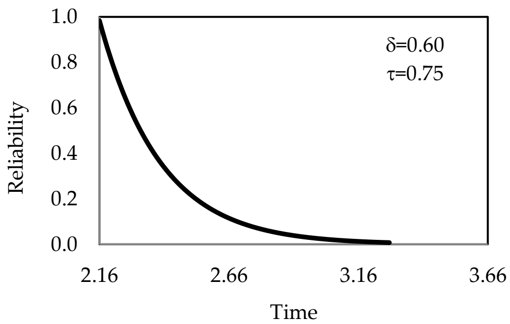

Figure 27: *Reliability versus time with $\delta = 0.60$, $\tau = 0.75$ for (2, 4) system.*

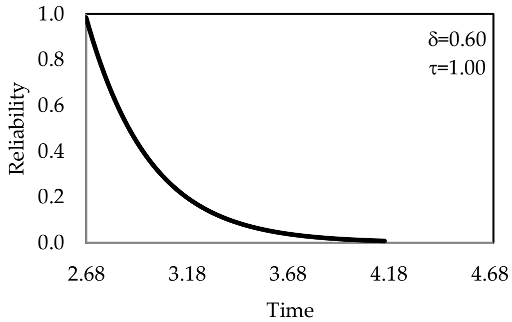

Figure 28: *Reliability versus time with $\delta = 0.60$, $\tau = 1$ for (2, 4) system.*

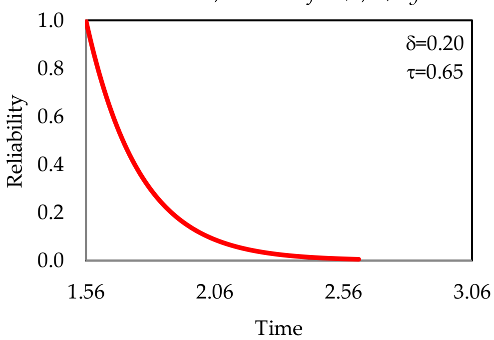

Figure 29: *Reliability versus time with $\delta = 0.2, \tau = 0.65$ for (2, 4) system.*

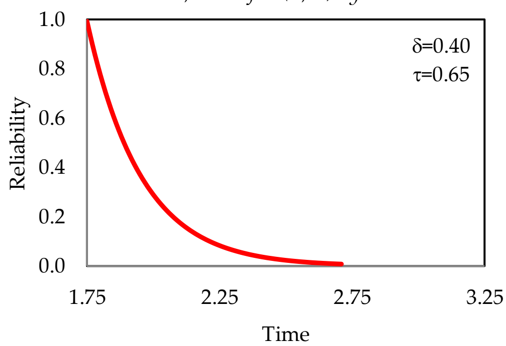

Figure 30: *Reliability versus time with $\delta = 0.40, \tau = 0.65$ for (2, 4) system.*

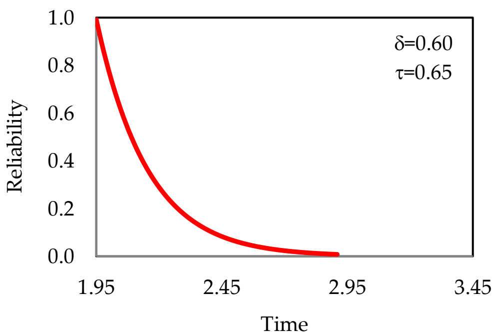

Figure 31: *Reliability versus time with $\delta = 0.6, \tau = 0.65$ for (2, 4) system.*

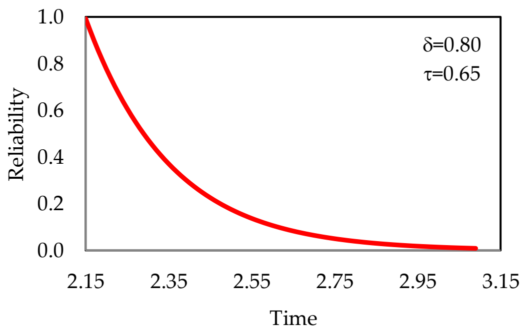

Figure 32: *Reliability versus time with $\delta = 0.8, \tau = 0.65$ for (2, 4) system.*

From table 7 and 8, it can be observed that for sequential (2, 4) system's reliability declines with increasing time and is also clear that time increases along with location and scale parameters.

From table 1,2,3,4,5,6,7,8 and figures, it is clear that as time increases, the different models of sequential (k, n) system's reliabilty decreases. It is also clear that time increases with increase in location and scale parameters. The results of numerical illustration clearly show that.

6. Conclusion

In this paper, different models of Sequential (k, n) system having exponential/gamma distribution with location and scale parameters are considered. The reliability function for different models of sequential (k, n) system's reliability decreases have been determined. Based on the findings in Table 1, 2, 3, 4, 5, 6, 7 & 8 and figures, we note the following

- By shifting the location and scale parameters, the reliability for different time are estimated for the suggested systems. As expected, the reliability decreases as the time increases.
- By monitoring reliability measures through shifting of parameters we can plan updates or patches to prevent system failures.
- It would be of interest to emulate this work for generalised gamma distribution and to extend this work for other continuous distributions, including Weibull distributions and Pareto distribution.

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