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RELIABILITY ANALYSIS OF C-SECTION WHERE STRENGTH AND SHEAR STRESS ARE NORMALLY DISTRIBUTED

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Abstract

The failure of a component depends on many parameters, such as complexity, time, design, reliability of components, and operating conditions. If failure depends on the stress of a component, such reliability models are called stress dependent models. There are many types of stresses that occur in the body, like tensile, compressive, shear, and bending. Shear stress develops in a body when a pair of opposite forces act across the section tangentially. In structural design, the choice of section shapes for different components is crucial for efficiency, strength, and stability. That's why C –sections are used as purlins. C-sections have a shape that allows for effective load distribution. In this paper, reliability analysis has been conducted over the C-section by applying load and finding the shear stress in the flange and web of C-section. It is observed from the computations that reliability decreases as the load and overall depth of the section increase. Reliability increases as the thickness and width of the web increase.

Keywords: Reliability, Normal distribution, Confidence interval, Shear stress, Hazard rate function.

I. Introduction

Reliability is the ability of the structure to meet the construction requirements set under specified conditions during the service life, according which it is designed. It refers to the capacity, serviceability and durability of construction and according to them different techniques of reliability can be defined.Many researchers have worked on stress-dependent models. Hong and Zhou [1] evaluated the reliability of RC columns. Val [2] studied the deterioration of strength of RC beams due to corrosion and its influence on beam reliability. Abubakar and Edrche [3] analyzed the reliability of simply supported steel beams using FORM. Breccolotti and Materazzi [4] described the structural reliability of eccentrically loaded sections in RC columns constructed by recycled aggregate concrete. Hariprasadetal.[5] analyzed the reliability for symmetrical columns with eccentric loading from a Lindley distribution. Satyanarayana et al[6] analyzed the reliability over I-Section of Beam due to Uniform Distribution of Load. Yakoob Pasha et al[7] described the reliability comparison of the shafts when shear stress follow the different distributions. Yakoob Pasha et al[8] analysed the reliability values using normal distribution. Wisconsin Electric Power Company [9] discussed the confidence intervals on operating basis earth quake and safe shut down earth quake.

II. Methodology

Reliability function is defined as the probability of success for the intended time t. $R(x) = 1 - F(x) = P(T > x)$ where T is a random variable denoting the failure time and X is random variable.The Hazard function $h(x)$is defined as the limit of the failure rate as the interval approaches zero. Thus, the hazard function is the instantaneous failure rate and is defined as $h(x) = \frac{f(x)}{R(x)} = \frac{f(x)}{1-F(x)}$where $f(x) = \frac{dF(x)}{dx}$. The reliability of an item is defined under stated operating and environmental conditions. This implies that any change in these conditions can effect. The failure rate of almost all components is stress dependent. A component can be influenced by more than one kind of stress. For such cases, a power function model is used $z(t) = h(t)\sigma_1^a\sigma_2^b$, where a, b are positive constants, $\sigma_1 and \sigma_2$are stress ratios for two different kinds of stresses, and z(t) is the failure rate at rated stress conditions.The normal distribution takes the well-known bell shape. This distribution is symmetrical bout its mean value. The probability density function for a normally distributed stress $\chi$and normally distributed strength ξ is given by

$$f_\chi(\chi) = \frac{1}{\sigma_\chi\sqrt{2\pi}} \exp\left[\frac{-1}{2}\left(\frac{\chi - \mu_\chi}{\sigma_\chi}\right)^2\right] \quad , for \ -\infty < \chi < \infty$$

$$f_\xi(\xi) = \frac{1}{\sigma_\xi\sqrt{2\pi}} \exp\left[\frac{-1}{2}\left(\frac{\xi - \mu_\xi}{\sigma_\xi}\right)^2\right] \quad , for \ -\infty < \xi < \infty$$

Where μ_χ = mean value of the stress
σ_χ = standard deviation of the stress
μ_ξ = mean value for the strength
σ_ξ = standard deviation of the strength
Let us define $y = \xi - \chi$. It is well known that the random variable $y$is normally distributed with mean $\mu_y = \mu_\xi - \mu_\chi$ and standard deviation $\sigma_y = \sqrt{\sigma^2{}_\xi + \sigma^2{}_\chi}$
The reliability Rcan be expressed in terms of $y$as

$$R = P(y > 0) = \frac{1}{\sigma_y\sqrt{2\pi}} \int_0^\infty \exp\left[\frac{-1}{2}\left(\frac{y - \mu_y}{\sigma_y}\right)^2\right] dy$$

$$let \ z = \frac{y - \mu_y}{\sigma_y}, \ then \ \sigma_y dz = dy$$

When y = 0, the lower limit of z is given by

And as y tends to ∞ the upper limit of z tends to ∞

$$z = \frac{0-\mu_y}{\sigma_y} = \frac{-\left(\mu_\xi - \mu_\chi\right)}{\sqrt{\sigma_\xi^2 + \sigma_\chi^2}}$$

And as y tends to ∞ the upper limit of z tends to ∞
Then the reliability is given by

$$R = \frac{1}{\sqrt{2\pi}} \int_{\frac{-\left(\mu_\xi - \mu_\chi\right)}{\sqrt{\sigma_\xi^2 + \sigma_\chi^2}}}^{\infty} \exp\left[\frac{-z^2}{2}\right] dz$$

The random variable $z = \frac{y-\mu_y}{\sigma_y}$ is the standard normal variable

Moment of inertia of the section about NA is

$$I = \frac{BD^3}{12} - \frac{bh^3}{12} = \frac{1}{12}\left(BD^3 - bh^3\right)$$

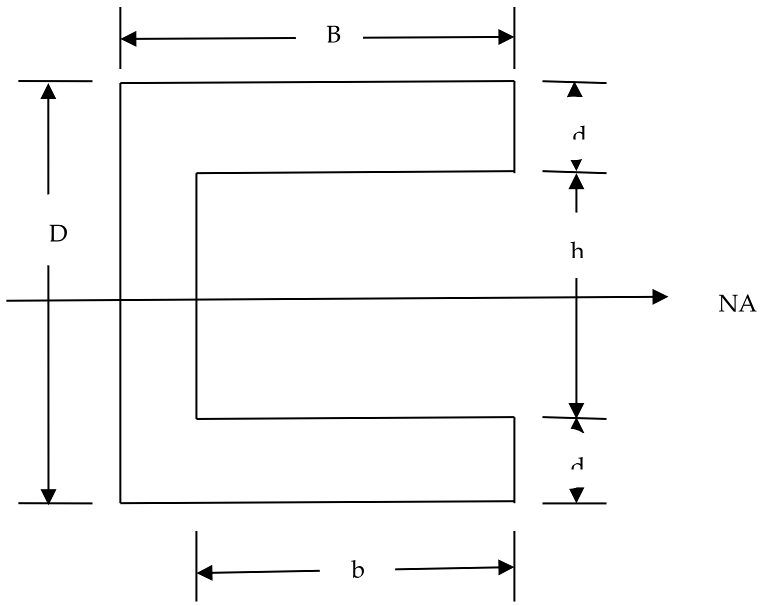

Figure 1:*C-section*

I. Shear stress at top of the Flange

The shear stress at top the flange in C section is

$$\tau_{flange} = \frac{FA\bar{y}}{IB} = \frac{F \times B \times d\left[\frac{D}{2} - \frac{d}{2}\right]}{IB} = \frac{Fd(D-d)}{2I}$$

let

$B = \mu_\xi = mean\ \ strength\ \ of\ Section$

$\tau_{flange} = \mu_\chi = mean\ \ stress\ \ of\ Section$

Then the reliability at top of the flange is

$$R = \frac{1}{\sqrt{2\pi}} \int_{\frac{-(\mu_\xi - \tau_{flange})}{\sqrt{\sigma_\xi^2 + \sigma_\chi^2}}}^{\infty} \exp\left[\frac{-z^2}{2}\right] dz$$

Reliability Computations

The reliability at the top of the flange is computed when B mean strength of the section varies for F=1000KN, D=150mm, d=20mm and depicted in the Figure 2.

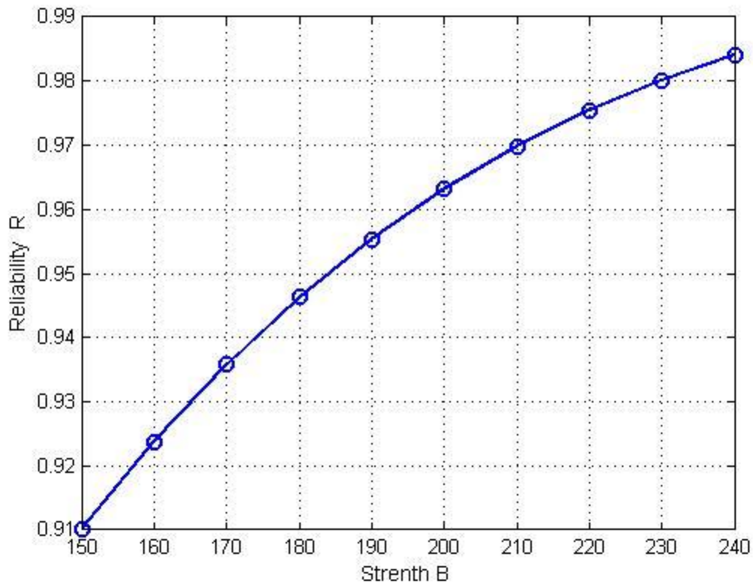

Figure 2: *Reliability Vs Strenth*

The reliability at the top of the flange is computed when d mean strength of the section varies for F=1000KN, B=150mm, D=150mmand depicted in the Figure3.

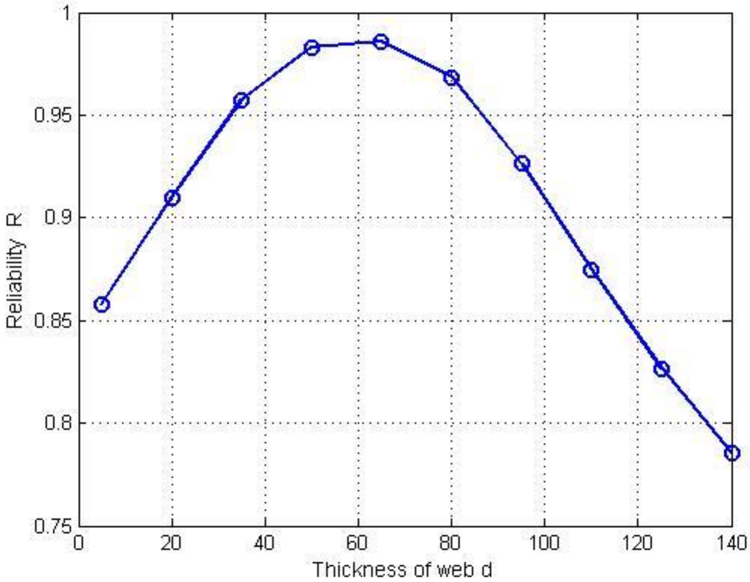

Figure 3: *Reliability Vs Thickness of web*

The reliability at the top of the flange is computed when F mean strength of the section varies for B=150mm, D=150mm, d=20mmand depicted in the Figure 4.

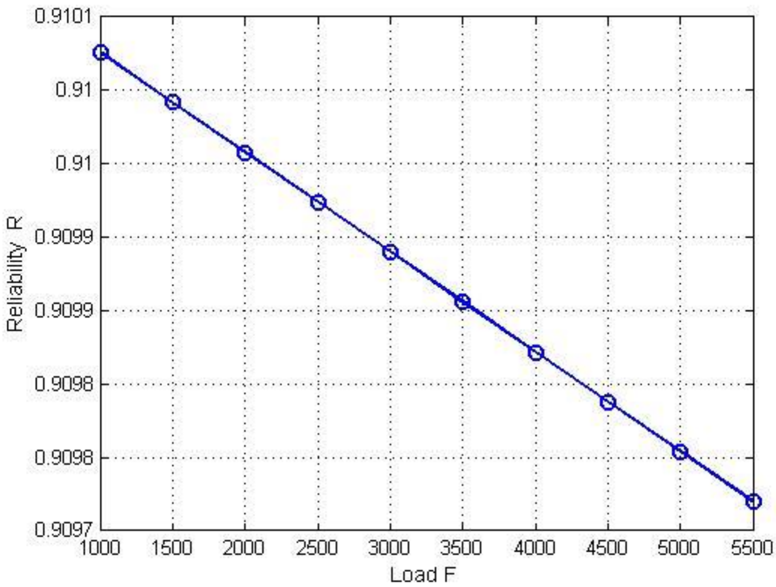

Figure 4: *Reliability Vs Load*

From figure 2, it is observed that if the strength increases then the reliability increases.From figure3, if the thickness of web increases then reliability increases to some extent and after that reliability decreases (i.e, from d=62mm).From figure4, if load increases then reliability decreases.

II. Shear stress at the Bottom of the Flange and Junction of Top of the Web

The Shear stress at the bottom of the flange and junction of top of the web in C section is

$$\tau_{web} = \frac{B}{d}\left[\frac{F \times B \times d \left[\frac{D}{2} - \frac{d}{2}\right]}{IB}\right]$$

$$\tau_{web} = \frac{FB(D-d)}{2I}$$

let

$B = \mu_{\xi} = mean\ strength\ of\ Section$

$\tau_{web} = \mu_{\chi} = mean\ stress\ of\ Section$

Then the reliability at top of the flange is

$$R = \frac{1}{\sqrt{2\pi}} \int_{\frac{-(\mu_{\xi} - \tau_{web})}{\sqrt{\sigma_{\xi}^2 + \sigma_{\chi}^2}}}^{\infty} \exp\left[\frac{-z^2}{2}\right] dz$$

Reliability Computations

The reliability at the top of the flange is computed when B mean strength of the section varies for F=1000KN, D=150mm, d=20mm and depicted in the Figure 5.

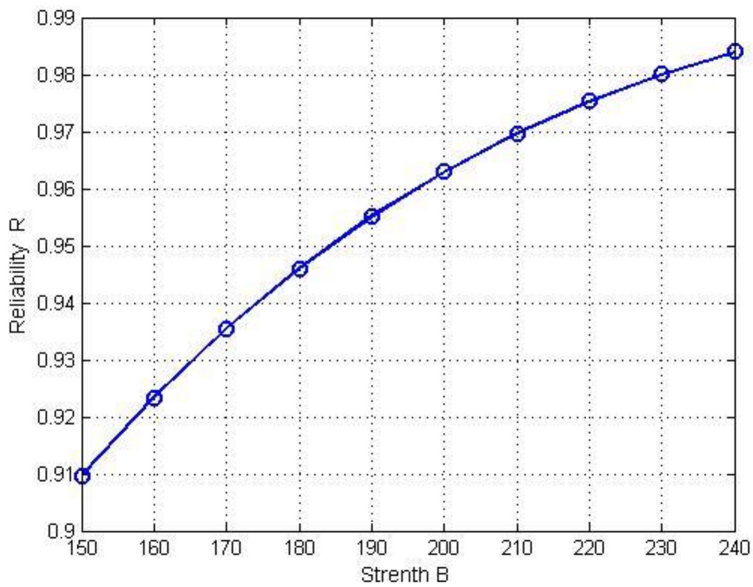

Figure 5: *Reliability Vs Strenth*

The reliability at the top of the flange is computed when d mean strength of the section varies for F=1000KN, B=150mm, D=150mmand depicted in the Figure 6.

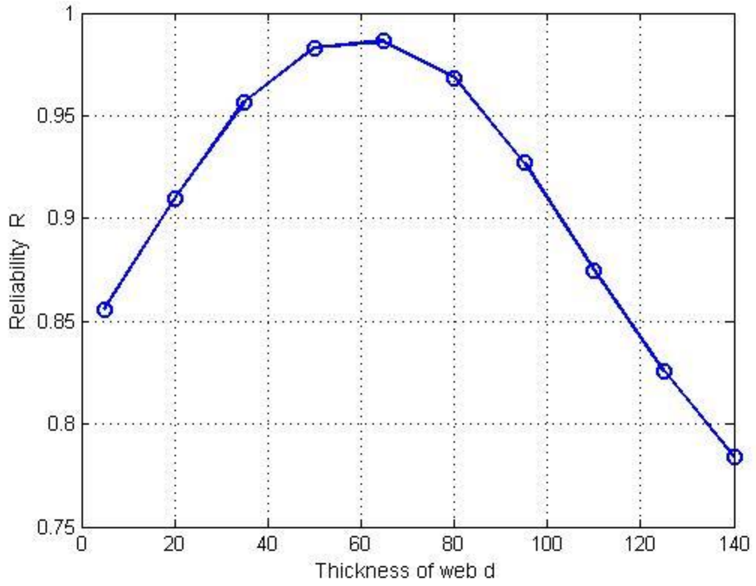

Figure 6: *Reliability Vs Thickness of web*

The reliability at the top of the flange is computed when F mean strength of the section varies for B=150mm, D=150mm, d=20mmand depicted in the Figure 7.

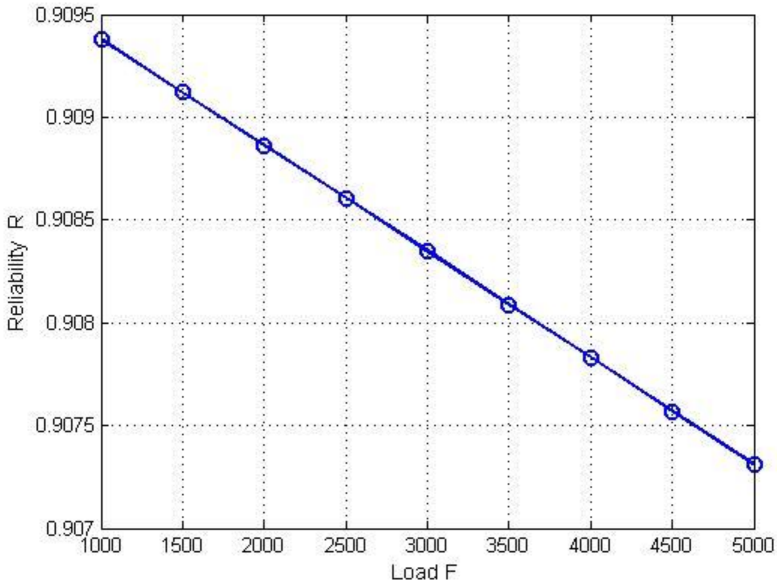

Figure 7: *Reliability Vs Load*

From figure 5, it is observed that if the strength increases then the reliability increases.From figure6, if the thickness of web increases then reliability increases to some extent and after that reliability decreases (i.efrom d=62mm).From figure7, if load increases then reliability decreases.

III. Shear stress at Neutral Axis

$$\tau_{\max} = \frac{F}{Id}\left[B \times d \times \left(\frac{d+h}{2}\right) + \left(d \times \frac{h}{2} \times \frac{h}{4}\right)\right]$$

$$\tau_{\max} = \frac{F\left[4B(d+h) + h^2\right]}{8I}$$

let

$B = \mu_\xi = \text{mean strength of Section}$

$\tau_{\max} = \mu_\chi = \text{mean stress of Section}$

Then the reliability at top of the flange is

$$R = \frac{1}{\sqrt{2\pi}} \int_{\frac{-(\mu_\xi - \tau_{\max})}{\sqrt{\sigma_\xi^2 + \sigma_\chi^2}}}^{\infty} \exp\left[\frac{-z^2}{2}\right] dz$$

Reliability Computations

The reliability at the top of the flange is computed when B mean strength of the section varies for F=1000KN, D=150mm, d=20mm and depicted in the Figure 8.

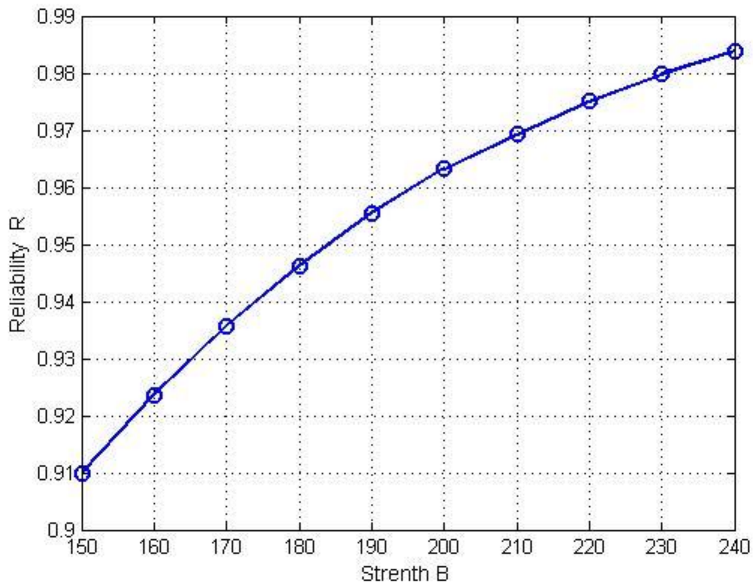

Figure 8: *Reliability Vs Strenth*

The reliability at the top of the flange is computed when d mean strength of the section varies for F=1000KN, B=150mm, D=150mmand depicted in the Figure 9.

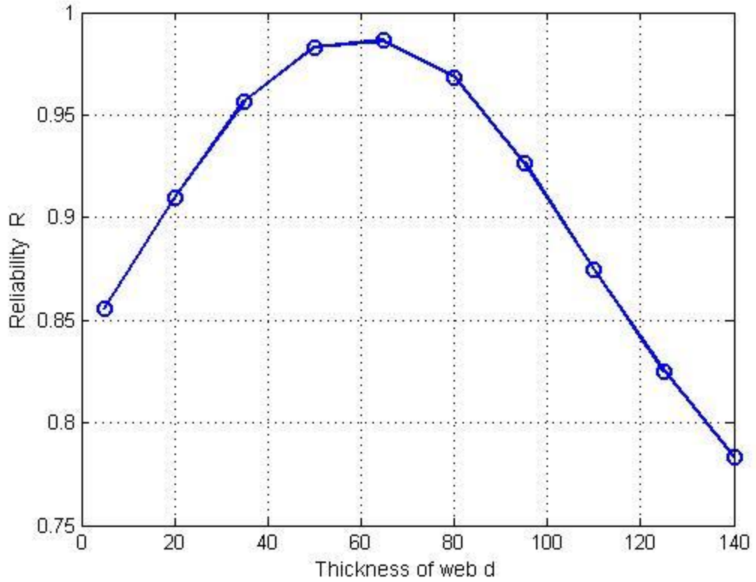

Figure 9: *Reliability Vs Thickness of web*

The reliability at the top of the flange is computed when F mean strength of the section varies for B=150mm, D=150mm, d=20mmand depicted in the Figure 10.

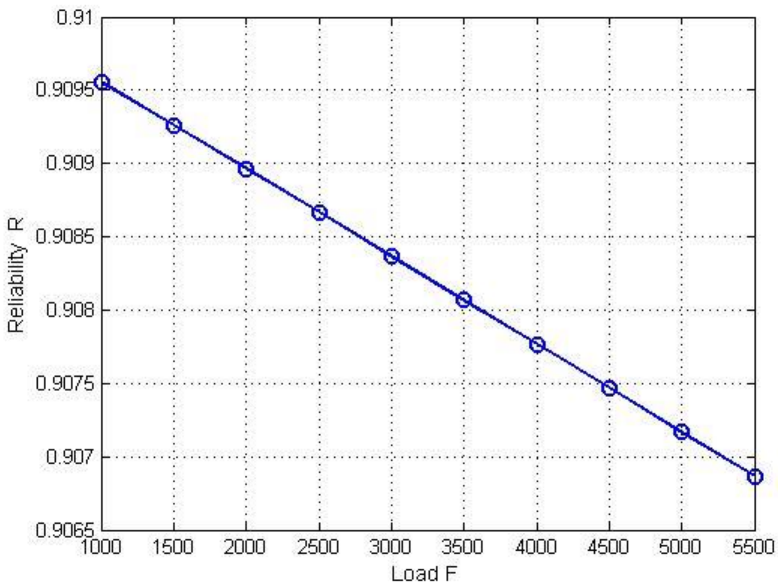

Figure 10: *Reliability Vs Load*

From figure 8, it is observed that if the strength increases then the reliability increases. From figure9, if the thickness of web increases then reliability increases to some extent and after that reliability decreases (i. e, from d=62mm). From figure10, if load increases then reliability decreases.

III. Confidence intervals

The confidence intervals for strength, thickness of web and load applied on C-section are used to observe the maximum reliability. For this, let us consider a large sample of size 40,50 and 50 for strength, thickness of web and load respectively. For this the strength values are considered from 50 mm to 440mm, thickness of web values are from 2mm to 149mm and loadfrom100 KN to 485200 KN. The confidence intervals will be find at different levels, 90%, 95%, 99% for both strength and load. The data never takes negative values and the distribution of data is skewed.

The statistical formula is $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

Here μ=μς mean of population strength, the sample mean is $\bar{x}$

Table 1

	Strength	Thickness of Web	Load
Sample Mean	245	75.5	242650
S.D	116.9045	103.9447	144316.1
Size	40	50	50

The confidence intervals for strength, at90% is (214.5934, 275.4065), at 95% is (208.7710, 281.2290), and at99% is (197.4031, 292.5968). The confidence intervals for thickness of web, at 90%is(51.3185, 99.6815),at 95% is(47.129, 103.8710),and at 99% is (37.6328, 113.3672).The confidence intervals for load, at 90% is (209076.5956,276223.4044), at 95% is (202647.6458, 282652.3542), and at 99% is (189993.8399, 295306.1601).

IV. Conclusion

The reliability is analysed at the top of the flange, bottom of the Flange and Junction of Top of the Web and at neutral axis. It increases with an increase in width at top of the flange, if the thickness of web increases then reliability increases to some extent and after that reliability decreases. The reliability decreases when the load Fincreases. The confidence intervals for both strength, thickness of web and load give a maximum reliability.

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