ESTIMATING THE POPULATION MEAN USING STRATIFIED DOUBLE UNIFIED RANKED SET SAMPLING FOR ASYMMETRIC DISTRIBUTIONS

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Abstract

In this study, we propose a novel sampling technique known as Stratified Unified Ranked Set Sampling (SDURSS) and evaluate its efficiency for estimating population means. SDURSS is designed to enhance the estimation accuracy by integrating concepts from ranked set sampling with stratified sampling. Our results demonstrate that the SDURSS estimator generally exhibits superior efficiency compared to SRS, particularly in complex distribution scenarios. While SDURSS often performs more efficiently than SSRS and SRSS, its performance relative to these methods varies depending on the specific distribution and sample size. In several cases, SDURSS outperforms SSRS and SRSS, highlighting its potential benefits in practical applications. The findings suggest that SDURSS is a promising alternative to traditional sampling methods, offering improved efficiency and potentially more accurate estimates of population means. This research underscores the value of exploring advanced sampling techniques to enhance statistical estimation, particularly in scenarios involving asymmetric distributions where traditional methods may be less effective.

Keywords: Simple random sampling, Ranked set sampling, Median Ranked Set Sampling, Unified ranked set sampling, Double Unified ranked set sampling, Stratified Double Unified ranked set sampling

I. Introduction

The ranked set sampling (RSS) method to estimate the population mean of average yields prosed by [1]. Later, RSS was developed and modified by many authors to estimate the population parameters. The mathematical proof for RSS. They proved that the sample mean based on RSS is an unbiased estimator of the population mean, which gave smaller variance than the sample mean based on a simple random sample (SRS) with the same sample size provided [2]. The variance of the sample mean based on RSS is less than or equal to that of the SRS, whether or not there are errors in ranking demonstrated by [3]. The RSS method of stratified ranked set sampling (SRSS) suggested by [4]. Some nonparametric tests for assessing the assumption of perfect ranking in RSS and powerful rank tests for perfect rankings proposed by [5-9], a Unified ranked sampling (URSS) suggested by [9-10]. RSS is called Double ranked set sampling (DRSS) developed by [1]. The RSS method is efficiency increasing the number of set and the number of cycles.

This study aims to propose the Stratified Double Unified Ranked Set Sampling (SDURSS) for estimating the population mean of asymmetric distributions and to study the efficiency of the empirical mean estimator based on SDURSS. Estimators in literature.

II. Materials and methods

I. Simple Random Sampling (SRS)

SRS is a method of selecting units out of units such that every one of the distinct samples has an equal chance of being drawn.

II. Stratified Sampling

In the stratified sampling method, the population of units is divided into non-overlapping sub-groups known as strata each stratum has units, respectively, such that For full benefit from stratification, the size of the hth strata, denoted by for, must be known. Then the samples are drawn independently from each stratum, producing sample sizes denoted by , such that the total sample size is If a simple random sample is taken from each stratum, the whole procedure is known as a stratified simple random sampling (SSRS).

III. Ranked Set Sampling (RSS)

RSS technique can be described as follows:
Step 1: Select samples of the SRS method from the population of interest.
Step 2: Allocate the selected units as randomly as possible into sets, each of size.
Step 3: Rank the units in each set with respect to the variable of interest.
Step 4: Choose a sample by taking the smallest ranked unit in the first set, the second smallest ranked unit in the second set, continue the process until the largest ranked unit is selected from the last set. Then the taken samples are measured the variable of interest.
Step 5: Repeat step 1 through step 4 for cycles to draw the RSS sample of size. [12]

Example 1: Let $(m = 7, r = 1)$ be

$$\begin{bmatrix} X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17} \\ X_{21}, X_{22}, X_{23}, X_{24}, X_{25}, X_{26}, X_{27} \\ X_{31}, X_{32}, X_{33}, X_{34}, X_{35}, X_{36}, X_{37} \\ X_{41}, X_{42}, X_{43}, X_{44}, X_{45}, X_{46}, X_{47} \\ X_{51}, X_{52}, X_{53}, X_{54}, X_{55}, X_{56}, X_{57} \\ X_{61}, X_{62}, X_{63}, X_{64}, X_{65}, X_{66}, X_{67} \\ X_{71}, X_{72}, X_{73}, X_{74}, X_{75}, X_{76}, X_{77} \end{bmatrix}$$

Then the measured RSS units are $X_{11}, X_{22}, X_{33}, X_{44}, X_{55}, X_{66}, X_{77}$

The empirical mean estimator of RSS is given by

$$\bar{X}_{RSS} = \frac{1}{mr} \sum_{j=1}^{r} \sum_{i=1}^{m} X_{(i:m)j}$$

and variance can be estimated by

$$Var\left(\bar{X}_{RSS}\right)=\frac{\sigma^2}{mr}-\frac{1}{m^2 r}\sum_{i=1}^{m}\left(\mu_{(i:m)j}-\mu\right)^2 .$$

Also

$$\mu=\frac{1}{mr}\sum_{j=1}^{r}\sum_{i=1}^{m}\mu_{(i:m)j}$$

Considerations:

1. Note: $m=$ set size, $r=$number of cycles)times(, $n=$sample of size(
2. The RSS use for Population infinite.

IV. Unified Ranked Set Sampling (URSS)

URSS technique can be described as follows:

Step 1: Use a SRS method to select m^2 units from the population of interest and rank them with respect to the variable of interest.

Step 2: Select the sample units for measurement as follow If m is an odd number, the ranked $\left(\frac{m+1}{2}+(i-1)m\right)$ units will be selected for $i=1,2,\cdots,m$. On the other hands, if m is an even number, divide the sample unit into 2 sections from size of the sample unit m^2. Where sections 1 select $\left(\frac{m}{2}+(i-1)m\right)$ units and sections 2 select $\left(\left(\frac{m}{2}+1\right)+(i-1)m\right)$units will be selected, for $i=1,2,\cdots,m$

Step 3: Repeat steps 1 and 2 for r cycles) for $j=1,2,\cdots,r$ (to draw the URSS of size $n=mr$

Define $x_{[i]j}$ be the URSS sampled unit of the i^{th} rank from the j^{th} cycle, where $i=1,2,\cdots,m$ and $j=1,2,\cdots,r$. [9-10]

)even number(

Example 2: Consider the case of $(m=6,r=1)$. Draw a simple random sample of size $m^2=6^2=36$ units as

$X_{11},X_{12},X_{13},X_{14},X_{15},X_{16},X_{17},X_{18},X_{19},X_{10},X_{11},X_{12},X_{13},X_{14},X_{15},X_{16},X_{17},X_{18},$
$X_{19},X_{20},X_{21},X_{22},X_{23},X_{24},X_{25},X_{26},X_{27},X_{28},X_{29},X_{30},X_{31},X_{32},X_{33},X_{34},X_{35},X_{36}$

From the size of the sample unit m^2, divide the sample unit into 2 sections

Where section 1, We select the sample unit from $\left(\frac{m}{2}+(i-1)m\right)$ for $i=1,2,\cdots,m$

$X_{11},X_{12},X_{13},X_{14},\boxed{X_{15}},X_{16},X_{17},X_{18},X_{19},X_{10},\boxed{X_{11}},X_{12},X_{13},X_{14},X_{15},X_{16},\boxed{X_{17}},X_{18}$

Where section 2, We select the sample unit from $\left(\left(\frac{m}{2}+1\right)+(i-1)m\right)$ for $i=1,2,\cdots,m$

$X_{19},X_{20},X_{21},X_{22},X_{23},\boxed{X_{24}},X_{25},X_{26},X_{27},X_{28},X_{29},\boxed{X_{30}},X_{31},X_{32},X_{33},X_{34},X_{35},\boxed{X_{36}}$

Let $X_{13},X_{19},X_{15},X_{22},X_{28},X_{34}$ is DURSS of size 6

)odd number(

Example 3: Consider the case of $(m=7,r=1)$. Draw a simple random sample of size $m^2=7^2=49$ units as

$X_{11},X_{12},X_{13},X_{14},X_{15},X_{16},X_{17},X_{18},X_{19},X_{10},X_{11},X_{12},X_{13},X_{14},X_{15},X_{16},X_{17},X_{18},$
$X_{19},X_{20},X_{21},X_{22},X_{23},X_{24},X_{25},X_{26},X_{27},X_{28},X_{29},X_{30},X_{31},X_{32},X_{33},X_{34},X_{35},$
$X_{36},X_{37},X_{38},X_{39},X_{40},X_{41},X_{42},X_{43},X_{44},X_{45},X_{46},X_{47},X_{48},X_{49}$

We select the sample unit from $\left(\frac{m+1}{2} + (i-1)m \right)$ for $i = 1, 2, \cdots, m$

$X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17}, X_{18}, X_{19}, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{17}, X_{18},$
$X_{19}, X_{20}, X_{21}, X_{22}, X_{23}, X_{24}, X_{25}, X_{26}, X_{27}, X_{28}, X_{29}, X_{30}, X_{31}, X_{32}, X_{33}, X_{34}, X_{35},$
$X_{36}, X_{37}, X_{38}, X_{39}, X_{40}, X_{41}, X_{42}, X_{43}, X_{44}, X_{45}, X_{46}, X_{47}, X_{48}, X_{49}$

Let $X_{14}, X_{11}, X_{18}, X_{25}, X_{32}, X_{39}, X_{46}$ is URSS of size 7

V. Double Unified Ranked Set Sampling (DURSS)

In research, the DURSS method is applied from the [11] follows:

Step 1: Use a SRS method to identify m^3 elements from the target population and divide these elements randomly into m sets each of size m^2 elements.

Step 2: Use the usual URSS procedure on each set to obtain m ranked set samples of size m each.

Step 3: Apply the URSS procedure again on step)2(to obtain a DURSS of size m.

The procedure is illustrated for the case of even and odd in the following example.

)even number(

Example 4: Consider the case of $(m = 6, r = 1)$. Draw a simple random sample of size $m^3 = 6^3 = 216$ elements)6 sets of size 36 each(. Assume the elements are

$$X^{(1)}_{(1)}, X^{(1)}_{(2)}, X^{(1)}_{(3)}, \cdots, X^{(1)}_{(36)}, X^{(2)}_{(1)}, X^{(2)}_{(2)}, X^{(2)}_{(3)}, \cdots, X^{(2)}_{(36)}, X^{(3)}_{(1)}, X^{(3)}_{(2)}, X^{(3)}_{(3)}, \cdots, X^{(3)}_{(36)},$$
$$X^{(4)}_{(1)}, X^{(4)}_{(2)}, X^{(4)}_{(3)}, \cdots, X^{(4)}_{(36)}, X^{(5)}_{(1)}, X^{(5)}_{(2)}, X^{(5)}_{(3)}, \cdots, X^{(5)}_{(36)}, X^{(6)}_{(1)}, X^{(6)}_{(2)}, X^{(6)}_{(3)}, \cdots, X^{(6)}_{(36)}$$

After ranking the elements of each set obtain 6 ranked set samples of size 6 each) $m^2 = 6^2 = 36$ (.

$$\begin{bmatrix} X^{(1)}_{(1)}, X^{(1)}_{(2)}, X^{(1)}_{(3)}, X^{(1)}_{(4)}, X^{(1)}_{(5)}, X^{(1)}_{(6)}, X^{(1)}_{(7)}, X^{(1)}_{(8)}, X^{(1)}_{(9)}, X^{(1)}_{(10)}, X^{(1)}_{(11)}, X^{(1)}_{(12)}, X^{(1)}_{(13)}, X^{(1)}_{(14)}, X^{(1)}_{(15)}, X^{(1)}_{(16)}, X^{(1)}_{(17)}, X^{(1)}_{(18)}, \\ X^{(1)}_{(19)}, X^{(1)}_{(20)}, X^{(1)}_{(21)}, X^{(1)}_{(22)}, X^{(1)}_{(23)}, X^{(1)}_{(24)}, X^{(1)}_{(25)}, X^{(1)}_{(26)}, X^{(1)}_{(27)}, X^{(1)}_{(28)}, X^{(1)}_{(29)}, X^{(1)}_{(30)}, X^{(1)}_{(31)}, X^{(1)}_{(32)}, X^{(1)}_{(33)}, X^{(1)}_{(34)}, X^{(1)}_{(35)}, X^{(1)}_{(36)} \end{bmatrix},$$

$$\begin{bmatrix} X^{(2)}_{(1)}, X^{(2)}_{(2)}, X^{(2)}_{(3)}, X^{(2)}_{(4)}, X^{(2)}_{(5)}, X^{(2)}_{(6)}, X^{(2)}_{(7)}, X^{(2)}_{(8)}, X^{(2)}_{(9)}, X^{(2)}_{(10)}, X^{(2)}_{(11)}, X^{(2)}_{(12)}, X^{(2)}_{(13)}, X^{(2)}_{(14)}, X^{(2)}_{(15)}, X^{(2)}_{(16)}, X^{(2)}_{(17)}, X^{(2)}_{(18)}, \\ X^{(2)}_{(19)}, X^{(2)}_{(20)}, X^{(2)}_{(21)}, X^{(2)}_{(22)}, X^{(2)}_{(23)}, X^{(2)}_{(24)}, X^{(2)}_{(25)}, X^{(2)}_{(26)}, X^{(2)}_{(27)}, X^{(2)}_{(28)}, X^{(2)}_{(29)}, X^{(2)}_{(30)}, X^{(2)}_{(31)}, X^{(2)}_{(32)}, X^{(2)}_{(33)}, X^{(2)}_{(34)}, X^{(2)}_{(35)}, X^{(2)}_{(36)} \end{bmatrix},$$

$$\begin{bmatrix} X^{(3)}_{(1)}, X^{(3)}_{(2)}, X^{(3)}_{(3)}, X^{(3)}_{(4)}, X^{(1)}_{(5)}, X^{(3)}_{(6)}, X^{(3)}_{(7)}, X^{(3)}_{(8)}, X^{(3)}_{(9)}, X^{(3)}_{(10)}, X^{(3)}_{(11)}, X^{(3)}_{(12)}, X^{(3)}_{(13)}, X^{(3)}_{(14)}, X^{(3)}_{(15)}, X^{(3)}_{(16)}, X^{(3)}_{(17)}, X^{(3)}_{(18)}, \\ X^{(3)}_{(19)}, X^{(3)}_{(20)}, X^{(3)}_{(21)}, X^{(3)}_{(22)}, X^{(3)}_{(23)}, X^{(3)}_{(24)}, X^{(3)}_{(25)}, X^{(3)}_{(26)}, X^{(3)}_{(27)}, X^{(3)}_{(28)}, X^{(3)}_{(29)}, X^{(3)}_{(30)}, X^{(3)}_{(31)}, X^{(3)}_{(32)}, X^{(3)}_{(33)}, X^{(3)}_{(34)}, X^{(3)}_{(35)}, X^{(3)}_{(36)} \end{bmatrix},$$

$$\begin{bmatrix} X^{(4)}_{(1)}, X^{(4)}_{(2)}, X^{(4)}_{(3)}, X^{(4)}_{(4)}, X^{(4)}_{(5)}, X^{(4)}_{(6)}, X^{(4)}_{(7)}, X^{(4)}_{(8)}, X^{(4)}_{(9)}, X^{(4)}_{(10)}, X^{(4)}_{(11)}, X^{(4)}_{(12)}, X^{(4)}_{(13)}, X^{(4)}_{(14)}, X^{(4)}_{(15)}, X^{(4)}_{(16)}, X^{(4)}_{(17)}, X^{(4)}_{(18)}, \\ X^{(4)}_{(19)}, X^{(4)}_{(20)}, X^{(4)}_{(21)}, X^{(4)}_{(22)}, X^{(4)}_{(23)}, X^{(4)}_{(24)}, X^{(4)}_{(25)}, X^{(4)}_{(26)}, X^{(4)}_{(27)}, X^{(4)}_{(28)}, X^{(4)}_{(29)}, X^{(4)}_{(30)}, X^{(4)}_{(31)}, X^{(4)}_{(32)}, X^{(4)}_{(33)}, X^{(4)}_{(34)}, X^{(4)}_{(35)}, X^{(4)}_{(36)} \end{bmatrix},$$

$$\begin{bmatrix} X^{(5)}_{(1)}, X^{(5)}_{(2)}, X^{(5)}_{(3)}, X^{(5)}_{(4)}, X^{(5)}_{(5)}, X^{(5)}_{(6)}, X^{(5)}_{(7)}, X^{(5)}_{(8)}, X^{(5)}_{(9)}, X^{(5)}_{(10)}, X^{(5)}_{(11)}, X^{(5)}_{(12)}, X^{(5)}_{(13)}, X^{(5)}_{(14)}, X^{(5)}_{(15)}, X^{(5)}_{(16)}, X^{(5)}_{(17)}, X^{(5)}_{(18)}, \\ X^{(5)}_{(19)}, X^{(5)}_{(20)}, X^{(5)}_{(21)}, X^{(5)}_{(22)}, X^{(5)}_{(23)}, X^{(5)}_{(124)}, X^{(5)}_{(25)}, X^{(5)}_{(26)}, X^{(5)}_{(27)}, X^{(5)}_{(28)}, X^{(5)}_{(29)}, X^{(5)}_{(30)}, X^{(5)}_{(31)}, X^{(5)}_{(32)}, X^{(5)}_{(33)}, X^{(5)}_{(34)}, X^{(5)}_{(35)}, X^{(5)}_{(36)} \end{bmatrix}$$

and

$$\begin{bmatrix} X^{(6)}_{(1)}, X^{(6)}_{(2)}, X^{(6)}_{(3)}, X^{(6)}_{(4)}, X^{(6)}_{(5)}, X^{(6)}_{(6)}, X^{(6)}_{(7)}, X^{(6)}_{(8)}, X^{(6)}_{(9)}, X^{(6)}_{(10)}, X^{(6)}_{(11)}, X^{(6)}_{(12)}, X^{(6)}_{(13)}, X^{(6)}_{(14)}, X^{(6)}_{(15)}, X^{(6)}_{(16)}, X^{(6)}_{(17)}, X^{(6)}_{(18)}, \\ X^{(6)}_{(19)}, X^{(6)}_{(20)}, X^{(6)}_{(21)}, X^{(6)}_{(22)}, X^{(6)}_{(23)}, X^{(6)}_{(24)}, X^{(6)}_{(25)}, X^{(6)}_{(26)}, X^{(6)}_{(27)}, X^{(6)}_{(28)}, X^{(6)}_{(29)}, X^{(6)}_{(30)}, X^{(6)}_{(31)}, X^{(6)}_{(32)}, X^{(6)}_{(33)}, X^{(6)}_{(34)}, X^{(6)}_{(35)}, X^{(6)}_{(36)} \end{bmatrix}$$

We select the sample unit from the elements of each set obtain 6 ranked set samples of size 6 each as

$$\left[\begin{array}{l} X^{(1)}_{(1)}, X^{(1)}_{(2)}, \boxed{X^{(1)}_{(3)}}, X^{(1)}_{(4)}, X^{(1)}_{(5)}, X^{(1)}_{(6)}, X^{(1)}_{(7)}, X^{(1)}_{(8)}, \boxed{X^{(1)}_{(9)}}, X^{(1)}_{(10)}, X^{(1)}_{(11)}, X^{(1)}_{(12)}, X^{(1)}_{(13)}, X^{(1)}_{(14)}, \boxed{X^{(1)}_{(15)}}, X^{(1)}_{(16)}, X^{(1)}_{(17)}, X^{(1)}_{(18)}, \\ X^{(1)}_{(19)}, X^{(1)}_{(20)}, X^{(1)}_{(21)}, \boxed{X^{(1)}_{(22)}}, X^{(1)}_{(23)}, X^{(1)}_{(24)}, X^{(1)}_{(25)}, X^{(1)}_{(26)}, X^{(1)}_{(27)}, \boxed{X^{(1)}_{(28)}}, X^{(1)}_{(29)}, X^{(1)}_{(30)}, X^{(1)}_{(31)}, X^{(1)}_{(32)}, X^{(1)}_{(33)}, \boxed{X^{(1)}_{(34)}}, X^{(1)}_{(35)}, X^{(1)}_{(36)} \end{array}\right],$$

$$\left[\begin{array}{l} X^{(2)}_{(1)}, X^{(2)}_{(2)}, \boxed{X^{(2)}_{(3)}}, X^{(2)}_{(4)}, X^{(2)}_{(5)}, X^{(2)}_{(6)}, X^{(2)}_{(7)}, X^{(2)}_{(8)}, \boxed{X^{(2)}_{(9)}}, X^{(2)}_{(10)}, X^{(2)}_{(11)}, X^{(2)}_{(12)}, X^{(2)}_{(13)}, X^{(2)}_{(14)}, \boxed{X^{(2)}_{(15)}}, X^{(2)}_{(16)}, X^{(2)}_{(17)}, X^{(2)}_{(18)}, \\ X^{(2)}_{(19)}, X^{(2)}_{(20)}, X^{(2)}_{(21)}, \boxed{X^{(2)}_{(22)}}, X^{(2)}_{(23)}, X^{(2)}_{(24)}, X^{(2)}_{(25)}, X^{(2)}_{(26)}, X^{(2)}_{(27)}, \boxed{X^{(2)}_{(28)}}, X^{(2)}_{(29)}, X^{(2)}_{(30)}, X^{(2)}_{(31)}, X^{(2)}_{(32)}, X^{(2)}_{(33)}, \boxed{X^{(2)}_{(34)}}, X^{(2)}_{(35)}, X^{(2)}_{(36)} \end{array}\right],$$

$$\left[\begin{array}{l} X^{(3)}_{(1)}, X^{(3)}_{(2)}, \boxed{X^{(3)}_{(3)}}, X^{(3)}_{(4)}, X^{(1)}_{(5)}, X^{(3)}_{(6)}, X^{(3)}_{(7)}, X^{(3)}_{(8)}, \boxed{X^{(3)}_{(9)}}, X^{(3)}_{(10)}, X^{(3)}_{(11)}, X^{(3)}_{(12)}, X^{(3)}_{(13)}, X^{(3)}_{(14)}, \boxed{X^{(3)}_{(15)}}, X^{(3)}_{(16)}, X^{(3)}_{(17)}, X^{(3)}_{(18)}, \\ X^{(3)}_{(19)}, X^{(3)}_{(20)}, X^{(3)}_{(21)}, \boxed{X^{(3)}_{(22)}}, X^{(3)}_{(23)}, X^{(3)}_{(24)}, X^{(3)}_{(25)}, X^{(3)}_{(26)}, X^{(3)}_{(27)}, \boxed{X^{(3)}_{(28)}}, X^{(3)}_{(29)}, X^{(3)}_{(30)}, X^{(3)}_{(31)}, X^{(3)}_{(32)}, X^{(3)}_{(33)}, \boxed{X^{(3)}_{(34)}}, X^{(3)}_{(35)}, X^{(3)}_{(36)} \end{array}\right],$$

$$\left[\begin{array}{l} X^{(4)}_{(1)}, X^{(4)}_{(2)}, \boxed{X^{(4)}_{(3)}}, X^{(4)}_{(4)}, X^{(4)}_{(5)}, X^{(4)}_{(6)}, X^{(4)}_{(7)}, X^{(4)}_{(8)}, \boxed{X^{(4)}_{(9)}}, X^{(4)}_{(10)}, X^{(4)}_{(11)}, X^{(4)}_{(12)}, X^{(4)}_{(13)}, X^{(4)}_{(14)}, \boxed{X^{(4)}_{(15)}}, X^{(4)}_{(16)}, X^{(4)}_{(17)}, X^{(4)}_{(18)}, \\ X^{(4)}_{(19)}, X^{(4)}_{(20)}, X^{(4)}_{(21)}, \boxed{X^{(4)}_{(22)}}, X^{(4)}_{(23)}, X^{(4)}_{(24)}, X^{(4)}_{(25)}, X^{(4)}_{(26)}, X^{(4)}_{(27)}, \boxed{X^{(4)}_{(28)}}, X^{(4)}_{(29)}, X^{(4)}_{(30)}, X^{(4)}_{(31)}, X^{(4)}_{(32)}, X^{(4)}_{(33)}, \boxed{X^{(4)}_{(34)}}, X^{(4)}_{(35)}, X^{(4)}_{(36)} \end{array}\right],$$

$$\left[\begin{array}{l} X^{(5)}_{(1)}, X^{(5)}_{(2)}, \boxed{X^{(5)}_{(3)}}, X^{(5)}_{(4)}, X^{(5)}_{(5)}, X^{(5)}_{(6)}, X^{(5)}_{(7)}, X^{(5)}_{(8)}, \boxed{X^{(5)}_{(9)}}, X^{(5)}_{(10)}, X^{(5)}_{(11)}, X^{(5)}_{(12)}, X^{(5)}_{(13)}, X^{(5)}_{(14)}, \boxed{X^{(5)}_{(15)}}, X^{(5)}_{(16)}, X^{(5)}_{(17)}, X^{(5)}_{(18)}, \\ X^{(5)}_{(19)}, X^{(5)}_{(20)}, X^{(5)}_{(21)}, \boxed{X^{(5)}_{(22)}}, X^{(5)}_{(23)}, X^{(5)}_{(124)}, X^{(5)}_{(25)}, X^{(5)}_{(26)}, X^{(5)}_{(27)}, \boxed{X^{(5)}_{(28)}}, X^{(5)}_{(29)}, X^{(5)}_{(30)}, X^{(5)}_{(31)}, X^{(5)}_{(32)}, X^{(5)}_{(33)}, \boxed{X^{(5)}_{(34)}}, X^{(5)}_{(35)}, X^{(5)}_{(36)} \end{array}\right]$$

and

$$\left[\begin{array}{l} X^{(6)}_{(1)}, X^{(6)}_{(2)}, \boxed{X^{(6)}_{(3)}}, X^{(6)}_{(4)}, X^{(6)}_{(5)}, X^{(6)}_{(6)}, X^{(6)}_{(7)}, X^{(6)}_{(8)}, \boxed{X^{(6)}_{(9)}}, X^{(6)}_{(10)}, X^{(6)}_{(11)}, X^{(6)}_{(12)}, X^{(6)}_{(13)}, X^{(6)}_{(14)}, \boxed{X^{(6)}_{(15)}}, X^{(6)}_{(16)}, X^{(6)}_{(17)}, X^{(6)}_{(18)}, \\ X^{(6)}_{(19)}, X^{(6)}_{(20)}, X^{(6)}_{(21)}, \boxed{X^{(6)}_{(22)}}, X^{(6)}_{(23)}, X^{(6)}_{(24)}, X^{(6)}_{(25)}, X^{(6)}_{(26)}, X^{(6)}_{(27)}, \boxed{X^{(6)}_{(28)}}, X^{(6)}_{(29)}, X^{(6)}_{(30)}, X^{(6)}_{(31)}, X^{(6)}_{(32)}, X^{(6)}_{(33)}, \boxed{X^{(6)}_{(34)}}, X^{(6)}_{(35)}, X^{(6)}_{(36)} \end{array}\right]$$

So, we have 6 DURSS

$$\begin{array}{l} X^{(1)}_{(3)}, X^{(1)}_{(9)}, X^{(1)}_{(15)}, X^{(1)}_{(22)}, X^{(1)}_{(28)}, X^{(1)}_{(34)}, \\ X^{(2)}_{(3)}, X^{(2)}_{(9)}, X^{(2)}_{(15)}, X^{(2)}_{(22)}, X^{(2)}_{(28)}, X^{(2)}_{(34)}, \\ X^{(3)}_{(3)}, X^{(3)}_{(9)}, X^{(3)}_{(15)}, X^{(3)}_{(22)}, X^{(3)}_{(28)}, X^{(3)}_{(34)}, \\ X^{(4)}_{(3)}, X^{(4)}_{(9)}, X^{(4)}_{(15)}, X^{(4)}_{(22)}, X^{(4)}_{(28)}, X^{(4)}_{(34)}, \\ X^{(5)}_{(3)}, X^{(5)}_{(9)}, X^{(5)}_{(15)}, X^{(5)}_{(22)}, X^{(5)}_{(28)}, X^{(5)}_{(34)}, \\ X^{(6)}_{(3)}, X^{(6)}_{(9)}, X^{(6)}_{(15)}, X^{(6)}_{(22)}, X^{(6)}_{(28)}, X^{(6)}_{(34)} \end{array}$$

We select the sample unit from 6 DURSS

$$\begin{array}{l} X^{(1)}_{(3)}, X^{(1)}_{(9)}, \boxed{X^{(1)}_{(15)}}, X^{(1)}_{(22)}, X^{(1)}_{(28)}, X^{(1)}_{(34)}, \\ X^{(2)}_{(3)}, X^{(2)}_{(9)}, \boxed{X^{(2)}_{(15)}}, X^{(2)}_{(22)}, X^{(2)}_{(28)}, X^{(2)}_{(34)}, \\ X^{(3)}_{(3)}, X^{(3)}_{(9)}, \boxed{X^{(3)}_{(15)}}, X^{(3)}_{(22)}, X^{(3)}_{(28)}, X^{(3)}_{(34)}, \\ X^{(4)}_{(3)}, X^{(4)}_{(9)}, X^{(4)}_{(15)}, \boxed{X^{(4)}_{(22)}}, X^{(4)}_{(28)}, X^{(4)}_{(34)}, \\ X^{(5)}_{(3)}, X^{(5)}_{(9)}, X^{(5)}_{(15)}, \boxed{X^{(5)}_{(22)}}, X^{(5)}_{(28)}, X^{(5)}_{(34)}, \\ X^{(6)}_{(3)}, X^{(6)}_{(9)}, X^{(6)}_{(15)}, \boxed{X^{(6)}_{(22)}}, X^{(6)}_{(28)}, X^{(6)}_{(34)} \end{array}$$

Let $X_{(15)}^{(1)}, X_{(15)}^{(2)}, X_{(15)}^{(3)}, X_{(22)}^{(4)}, X_{(22)}^{(5)}, X_{(22)}^{(6)}$ is DURSS of size 6

)odd number(

Example 5: Consider the case of $(m=7, r=1)$. Draw a simple random sample of size $m^3 = 7^3 = 343$ elements)7 sets of size 49 each(. Assume the elements are

$$X_{(1)}^{(1)}, X_{(2)}^{(1)}, X_{(3)}^{(1)}, \cdots, X_{(49)}^{(1)}, X_{(1)}^{(2)}, X_{(2)}^{(2)}, X_{(3)}^{(2)}, \cdots, X_{(49)}^{(2)}, X_{(1)}^{(3)}, X_{(2)}^{(3)}, X_{(3)}^{(3)}, \cdots, X_{(49)}^{(3)}, X_{(1)}^{(4)}, X_{(2)}^{(4)}, X_{(3)}^{(4)}, \cdots, X_{(49)}^{(4)},$$
$$X_{(1)}^{(5)}, X_{(2)}^{(5)}, X_{(3)}^{(5)}, \cdots, X_{(49)}^{(5)}, X_{(1)}^{(6)}, X_{(2)}^{(6)}, X_{(3)}^{(6)}, \cdots, X_{(49)}^{(6)}, X_{(1)}^{(7)}, X_{(2)}^{(7)}, X_{(3)}^{(7)}, \cdots, X_{(49)}^{(7)}$$

After ranking the elements of each set obtain 7 ranked set samples of size 7 each) $m^2 = 7^2 = 49$ (.

$$\left[\begin{array}{l} X_{(1)}^{(1)}, X_{(2)}^{(1)}, X_{(3)}^{(1)}, X_{(4)}^{(1)}, X_{(5)}^{(1)}, X_{(6)}^{(1)}, X_{(7)}^{(1)}, X_{(8)}^{(1)}, X_{(9)}^{(1)}, X_{(10)}^{(1)}, X_{(11)}^{(1)}, X_{(12)}^{(1)}, X_{(13)}^{(1)}, X_{(14)}^{(1)}, X_{(15)}^{(1)}, X_{(16)}^{(1)}, X_{(17)}^{(1)}, X_{(18)}^{(1)}, \\ X_{(19)}^{(1)}, X_{(20)}^{(1)}, X_{(21)}^{(1)}, X_{(22)}^{(1)}, X_{(23)}^{(1)}, X_{(24)}^{(1)}, X_{(25)}^{(1)}, X_{(26)}^{(1)}, X_{(27)}^{(1)}, X_{(28)}^{(1)}, X_{(29)}^{(1)}, X_{(30)}^{(1)}, X_{(31)}^{(1)}, X_{(32)}^{(1)}, X_{(33)}^{(1)}, X_{(34)}^{(1)}, X_{(35)}^{(1)}, X_{(36)}^{(1)}, \\ X_{(37)}^{(1)}, X_{(38)}^{(1)}, X_{(39)}^{(1)}, X_{(40)}^{(1)}, X_{(41)}^{(1)}, X_{(42)}^{(1)}, X_{(43)}^{(1)}, X_{(44)}^{(1)}, X_{(45)}^{(1)}, X_{(46)}^{(1)}, X_{(47)}^{(1)}, X_{(48)}^{(1)}, X_{(49)}^{(1)} \end{array}\right],$$

$$\left[\begin{array}{l} X_{(1)}^{(2)}, X_{(2)}^{(2)}, X_{(3)}^{(2)}, X_{(4)}^{(2)}, X_{(5)}^{(2)}, X_{(6)}^{(2)}, X_{(7)}^{(2)}, X_{(8)}^{(2)}, X_{(9)}^{(2)}, X_{(10)}^{(2)}, X_{(11)}^{(2)}, X_{(12)}^{(2)}, X_{(13)}^{(2)}, X_{(14)}^{(2)}, X_{(15)}^{(2)}, X_{(16)}^{(2)}, X_{(17)}^{(2)}, X_{(18)}^{(2)}, \\ X_{(19)}^{(2)}, X_{(20)}^{(2)}, X_{(21)}^{(2)}, X_{(22)}^{(2)}, X_{(23)}^{(2)}, X_{(24)}^{(2)}, X_{(25)}^{(2)}, X_{(26)}^{(2)}, X_{(27)}^{(2)}, X_{(28)}^{(2)}, X_{(29)}^{(2)}, X_{(30)}^{(2)}, X_{(31)}^{(2)}, X_{(32)}^{(2)}, X_{(33)}^{(2)}, X_{(34)}^{(2)}, X_{(35)}^{(2)}, X_{(36)}^{(2)}, \\ X_{(37)}^{(2)}, X_{(38)}^{(2)}, X_{(39)}^{(2)}, X_{(40)}^{(2)}, X_{(41)}^{(2)}, X_{(42)}^{(2)}, X_{(43)}^{(2)}, X_{(44)}^{(2)}, X_{(45)}^{(2)}, X_{(46)}^{(2)}, X_{(47)}^{(2)}, X_{(48)}^{(2)}, X_{(49)}^{(2)} \end{array}\right],$$

$$\left[\begin{array}{l} X_{(1)}^{(3)}, X_{(2)}^{(3)}, X_{(3)}^{(3)}, X_{(4)}^{(3)}, X_{(5)}^{(1)}, X_{(6)}^{(3)}, X_{(7)}^{(3)}, X_{(8)}^{(3)}, X_{(9)}^{(3)}, X_{(10)}^{(3)}, X_{(11)}^{(3)}, X_{(12)}^{(3)}, X_{(13)}^{(3)}, X_{(14)}^{(3)}, X_{(15)}^{(3)}, X_{(16)}^{(3)}, X_{(17)}^{(3)}, X_{(18)}^{(3)}, \\ X_{(19)}^{(3)}, X_{(20)}^{(3)}, X_{(21)}^{(3)}, X_{(22)}^{(3)}, X_{(23)}^{(3)}, X_{(24)}^{(3)}, X_{(25)}^{(3)}, X_{(26)}^{(3)}, X_{(27)}^{(3)}, X_{(28)}^{(3)}, X_{(29)}^{(3)}, X_{(30)}^{(3)}, X_{(31)}^{(3)}, X_{(32)}^{(3)}, X_{(33)}^{(3)}, X_{(34)}^{(3)}, X_{(35)}^{(3)}, X_{(36)}^{(3)}, \\ X_{(37)}^{(3)}, X_{(38)}^{(3)}, X_{(39)}^{(3)}, X_{(40)}^{(3)}, X_{(41)}^{(3)}, X_{(42)}^{(3)}, X_{(43)}^{(3)}, X_{(44)}^{(3)}, X_{(45)}^{(3)}, X_{(46)}^{(3)}, X_{(47)}^{(3)}, X_{(48)}^{(3)}, X_{(49)}^{(3)} \end{array}\right],$$

$$\left[\begin{array}{l} X_{(1)}^{(4)}, X_{(2)}^{(4)}, X_{(3)}^{(4)}, X_{(4)}^{(4)}, X_{(5)}^{(4)}, X_{(6)}^{(4)}, X_{(7)}^{(4)}, X_{(8)}^{(4)}, X_{(9)}^{(4)}, X_{(10)}^{(4)}, X_{(11)}^{(4)}, X_{(12)}^{(4)}, X_{(13)}^{(4)}, X_{(14)}^{(4)}, X_{(15)}^{(4)}, X_{(16)}^{(4)}, X_{(17)}^{(4)}, X_{(18)}^{(4)}, \\ X_{(19)}^{(4)}, X_{(20)}^{(4)}, X_{(21)}^{(4)}, X_{(22)}^{(4)}, X_{(23)}^{(4)}, X_{(24)}^{(4)}, X_{(25)}^{(4)}, X_{(26)}^{(4)}, X_{(27)}^{(4)}, X_{(28)}^{(4)}, X_{(29)}^{(4)}, X_{(30)}^{(4)}, X_{(31)}^{(4)}, X_{(32)}^{(4)}, X_{(33)}^{(4)}, X_{(34)}^{(4)}, X_{(35)}^{(4)}, X_{(36)}^{(4)}, \\ X_{(37)}^{(4)}, X_{(38)}^{(4)}, X_{(39)}^{(4)}, X_{(40)}^{(4)}, X_{(41)}^{(4)}, X_{(42)}^{(4)}, X_{(43)}^{(4)}, X_{(44)}^{(4)}, X_{(45)}^{(4)}, X_{(46)}^{(4)}, X_{(47)}^{(4)}, X_{(48)}^{(4)}, X_{(49)}^{(4)} \end{array}\right],$$

$$\left[\begin{array}{l} X_{(1)}^{(5)}, X_{(2)}^{(5)}, X_{(3)}^{(5)}, X_{(4)}^{(5)}, X_{(5)}^{(5)}, X_{(6)}^{(5)}, X_{(7)}^{(5)}, X_{(8)}^{(5)}, X_{(9)}^{(5)}, X_{(10)}^{(5)}, X_{(11)}^{(5)}, X_{(12)}^{(5)}, X_{(13)}^{(5)}, X_{(14)}^{(5)}, X_{(15)}^{(5)}, X_{(16)}^{(5)}, X_{(17)}^{(5)}, X_{(18)}^{(5)}, \\ X_{(19)}^{(5)}, X_{(20)}^{(5)}, X_{(21)}^{(5)}, X_{(22)}^{(5)}, X_{(23)}^{(5)}, X_{(124)}^{(5)}, X_{(25)}^{(5)}, X_{(26)}^{(5)}, X_{(27)}^{(5)}, X_{(28)}^{(5)}, X_{(29)}^{(5)}, X_{(30)}^{(5)}, X_{(31)}^{(5)}, X_{(32)}^{(5)}, X_{(33)}^{(5)}, X_{(34)}^{(5)}, X_{(35)}^{(5)}, X_{(36)}^{(5)}, \\ X_{(37)}^{(5)}, X_{(38)}^{(5)}, X_{(39)}^{(5)}, X_{(40)}^{(5)}, X_{(41)}^{(5)}, X_{(42)}^{(5)}, X_{(43)}^{(5)}, X_{(44)}^{(5)}, X_{(45)}^{(5)}, X_{(46)}^{(5)}, X_{(47)}^{(5)}, X_{(48)}^{(5)}, X_{(49)}^{(5)} \end{array}\right],$$

$$\left[\begin{array}{l} X_{(1)}^{(6)}, X_{(2)}^{(6)}, X_{(3)}^{(6)}, X_{(4)}^{(6)}, X_{(5)}^{(6)}, X_{(6)}^{(6)}, X_{(7)}^{(6)}, X_{(8)}^{(6)}, X_{(9)}^{(6)}, X_{(10)}^{(6)}, X_{(11)}^{(6)}, X_{(12)}^{(6)}, X_{(13)}^{(6)}, X_{(14)}^{(6)}, X_{(15)}^{(6)}, X_{(16)}^{(6)}, X_{(17)}^{(6)}, X_{(18)}^{(6)}, \\ X_{(19)}^{(6)}, X_{(20)}^{(6)}, X_{(21)}^{(6)}, X_{(22)}^{(6)}, X_{(23)}^{(6)}, X_{(24)}^{(6)}, X_{(25)}^{(6)}, X_{(26)}^{(6)}, X_{(27)}^{(6)}, X_{(28)}^{(6)}, X_{(29)}^{(6)}, X_{(30)}^{(6)}, X_{(31)}^{(6)}, X_{(32)}^{(6)}, X_{(33)}^{(6)}, X_{(34)}^{(6)}, X_{(35)}^{(6)}, X_{(36)}^{(6)}, \\ X_{(37)}^{(6)}, X_{(38)}^{(6)}, X_{(39)}^{(6)}, X_{(40)}^{(6)}, X_{(41)}^{(6)}, X_{(42)}^{(6)}, X_{(43)}^{(6)}, X_{(44)}^{(6)}, X_{(45)}^{(6)}, X_{(46)}^{(6)}, X_{(47)}^{(6)}, X_{(48)}^{(6)}, X_{(49)}^{(6)} \end{array}\right]$$

and

$$\left[\begin{array}{l} X_{(1)}^{(7)},X_{(2)}^{(7)},X_{(3)}^{(7)},X_{(4)}^{(7)},X_{(5)}^{(7)},X_{(6)}^{(7)},X_{(7)}^{(7)},X_{(8)}^{(7)},X_{(9)}^{(7)},X_{(10)}^{(7)},X_{(11)}^{(7)},X_{(12)}^{(7)},X_{(13)}^{(7)},X_{(14)}^{(7)},X_{(15)}^{(7)},X_{(16)}^{(7)},X_{(17)}^{(7)},X_{(18)}^{(7)}, \\ X_{(19)}^{(7)},X_{(20)}^{(7)},X_{(21)}^{(7)},X_{(22)}^{(7)},X_{(23)}^{(7)},X_{(24)}^{(7)},X_{(25)}^{(7)},X_{(26)}^{(7)},X_{(27)}^{(7)},X_{(28)}^{(7)},X_{(29)}^{(7)},X_{(30)}^{(7)},X_{(31)}^{(7)},X_{(32)}^{(7)},X_{(33)}^{(7)},X_{(34)}^{(7)},X_{(35)}^{(7)},X_{(36)}^{(7)}, \\ X_{(37)}^{(7)},X_{(38)}^{(7)},X_{(39)}^{(7)},X_{(40)}^{(7)},X_{(41)}^{(7)},X_{(42)}^{(7)},X_{(43)}^{(7)},X_{(44)}^{(7)},X_{(45)}^{(7)},X_{(46)}^{(7)},X_{(47)}^{(7)},X_{(48)}^{(7)},X_{(49)}^{(7)} \end{array}\right]$$

We select the sample unit from the elements of each set obtain 7 ranked set samples of size 7 each as

$$\left[\begin{array}{l} X_{(1)}^{(1)},X_{(2)}^{(1)},X_{(3)}^{(1)},\boxed{X_{(4)}^{(1)},}X_{(5)}^{(1)},X_{(6)}^{(1)},X_{(7)}^{(1)},X_{(8)}^{(1)},X_{(9)}^{(1)},X_{(10)}^{(1)},\boxed{X_{(11)}^{(1)},}X_{(12)}^{(1)},X_{(13)}^{(1)},X_{(14)}^{(1)},X_{(15)}^{(1)},X_{(16)}^{(1)},X_{(17)}^{(1)},\boxed{X_{(18)}^{(1)},} \\ X_{(19)}^{(1)},X_{(20)}^{(1)},X_{(21)}^{(1)},X_{(22)}^{(1)},X_{(23)}^{(1)},X_{(24)}^{(1)},\boxed{X_{(25)}^{(1)},}X_{(26)}^{(1)},X_{(27)}^{(1)},X_{(28)}^{(1)},X_{(29)}^{(1)},X_{(30)}^{(1)},X_{(31)}^{(1)},\boxed{X_{(32)}^{(1)},}X_{(33)}^{(1)},X_{(34)}^{(1)},X_{(35)}^{(1)},X_{(36)}^{(1)}, \\ X_{(37)}^{(1)},X_{(38)}^{(1)},\boxed{X_{(39)}^{(1)},}X_{(40)}^{(1)},X_{(41)}^{(1)},X_{(42)}^{(1)},X_{(43)}^{(1)},X_{(44)}^{(1)},X_{(45)}^{(1)},\boxed{X_{(46)}^{(1)},}X_{(47)}^{(1)},X_{(48)}^{(1)},X_{(49)}^{(1)} \end{array}\right],$$

$$\left[\begin{array}{l} X_{(1)}^{(2)},X_{(2)}^{(2)},X_{(3)}^{(2)},\boxed{X_{(4)}^{(2)},}X_{(5)}^{(2)},X_{(6)}^{(2)},X_{(7)}^{(2)},X_{(8)}^{(2)},X_{(9)}^{(2)},X_{(10)}^{(2)},\boxed{X_{(11)}^{(2)},}X_{(12)}^{(2)},X_{(13)}^{(2)},X_{(14)}^{(2)},X_{(15)}^{(2)},X_{(16)}^{(2)},X_{(17)}^{(2)},\boxed{X_{(18)}^{(2)},} \\ X_{(19)}^{(2)},X_{(20)}^{(2)},X_{(21)}^{(2)},X_{(22)}^{(2)},X_{(23)}^{(2)},X_{(24)}^{(2)},\boxed{X_{(25)}^{(2)},}X_{(26)}^{(2)},X_{(27)}^{(2)},X_{(28)}^{(2)},X_{(29)}^{(2)},X_{(30)}^{(2)},X_{(31)}^{(2)},\boxed{X_{(32)}^{(2)},}X_{(33)}^{(2)},X_{(34)}^{(2)},X_{(35)}^{(2)},X_{(36)}^{(2)}, \\ X_{(37)}^{(2)},X_{(38)}^{(2)},\boxed{X_{(39)}^{(2)},}X_{(40)}^{(2)},X_{(41)}^{(2)},X_{(42)}^{(2)},X_{(43)}^{(2)},X_{(44)}^{(2)},X_{(45)}^{(2)},\boxed{X_{(46)}^{(2)},}X_{(47)}^{(2)},X_{(48)}^{(2)},X_{(49)}^{(2)} \end{array}\right],$$

$$\left[\begin{array}{l} X_{(1)}^{(3)},X_{(2)}^{(3)},X_{(3)}^{(3)},\boxed{X_{(4)}^{(3)},}X_{(5)}^{(1)},X_{(6)}^{(3)},X_{(7)}^{(3)},X_{(8)}^{(3)},X_{(9)}^{(3)},X_{(10)}^{(3)},\boxed{X_{(11)}^{(3)},}X_{(12)}^{(3)},X_{(13)}^{(3)},X_{(14)}^{(3)},X_{(15)}^{(3)},X_{(16)}^{(3)},X_{(17)}^{(3)},\boxed{X_{(18)}^{(3)},} \\ X_{(19)}^{(3)},X_{(20)}^{(3)},X_{(21)}^{(3)},X_{(22)}^{(3)},X_{(23)}^{(3)},X_{(24)}^{(3)},\boxed{X_{(25)}^{(3)},}X_{(26)}^{(3)},X_{(27)}^{(3)},X_{(28)}^{(3)},X_{(29)}^{(3)},X_{(30)}^{(3)},X_{(31)}^{(3)},\boxed{X_{(32)}^{(3)},}X_{(33)}^{(3)},X_{(34)}^{(3)},X_{(35)}^{(3)},X_{(36)}^{(3)}, \\ X_{(37)}^{(3)},X_{(38)}^{(3)},\boxed{X_{(39)}^{(3)},}X_{(40)}^{(3)},X_{(41)}^{(3)},X_{(42)}^{(3)},X_{(43)}^{(3)},X_{(44)}^{(3)},X_{(45)}^{(3)},\boxed{X_{(46)}^{(3)},}X_{(47)}^{(3)},X_{(48)}^{(3)},X_{(49)}^{(3)} \end{array}\right],$$

$$\left[\begin{array}{l} X_{(1)}^{(4)},X_{(2)}^{(4)},X_{(3)}^{(4)},\boxed{X_{(4)}^{(4)},}X_{(5)}^{(4)},X_{(6)}^{(4)},X_{(7)}^{(4)},X_{(8)}^{(4)},X_{(9)}^{(4)},X_{(10)}^{(4)},\boxed{X_{(11)}^{(4)},}X_{(12)}^{(4)},X_{(13)}^{(4)},X_{(14)}^{(4)},X_{(15)}^{(4)},X_{(16)}^{(4)},X_{(17)}^{(4)},\boxed{X_{(18)}^{(4)},} \\ X_{(19)}^{(4)},X_{(20)}^{(4)},X_{(21)}^{(4)},X_{(22)}^{(4)},X_{(23)}^{(4)},X_{(24)}^{(4)},\boxed{X_{(25)}^{(4)},}X_{(26)}^{(4)},X_{(27)}^{(4)},X_{(28)}^{(4)},X_{(29)}^{(4)},X_{(30)}^{(4)},X_{(31)}^{(4)},\boxed{X_{(32)}^{(4)},}X_{(33)}^{(4)},X_{(34)}^{(4)},X_{(35)}^{(4)},X_{(36)}^{(4)}, \\ X_{(37)}^{(4)},X_{(38)}^{(4)},\boxed{X_{(39)}^{(4)},}X_{(40)}^{(4)},X_{(41)}^{(4)},X_{(42)}^{(4)},X_{(43)}^{(4)},X_{(44)}^{(4)},X_{(45)}^{(4)},\boxed{X_{(46)}^{(4)},}X_{(47)}^{(4)},X_{(48)}^{(4)},X_{(49)}^{(4)} \end{array}\right],$$

$$\left[\begin{array}{l} X_{(1)}^{(5)},X_{(2)}^{(5)},X_{(3)}^{(5)},\boxed{X_{(4)}^{(5)},}X_{(5)}^{(5)},X_{(6)}^{(5)},X_{(7)}^{(5)},X_{(8)}^{(5)},X_{(9)}^{(5)},X_{(10)}^{(5)},\boxed{X_{(11)}^{(5)},}X_{(12)}^{(5)},X_{(13)}^{(5)},X_{(14)}^{(5)},X_{(15)}^{(5)},X_{(16)}^{(5)},X_{(17)}^{(5)},\boxed{X_{(18)}^{(5)},} \\ X_{(19)}^{(5)},X_{(20)}^{(5)},X_{(21)}^{(5)},X_{(22)}^{(5)},X_{(23)}^{(5)},X_{(124)}^{(5)},\boxed{X_{(25)}^{(5)},}X_{(26)}^{(5)},X_{(27)}^{(5)},X_{(28)}^{(5)},X_{(29)}^{(5)},X_{(30)}^{(5)},X_{(31)}^{(5)},\boxed{X_{(32)}^{(5)},}X_{(33)}^{(5)},X_{(34)}^{(5)},X_{(35)}^{(5)},X_{(36)}^{(5)}, \\ X_{(37)}^{(5)},X_{(38)}^{(5)},\boxed{X_{(39)}^{(5)},}X_{(40)}^{(5)},X_{(41)}^{(5)},X_{(42)}^{(5)},X_{(43)}^{(5)},X_{(44)}^{(5)},X_{(45)}^{(5)},\boxed{X_{(46)}^{(5)},}X_{(47)}^{(5)},X_{(48)}^{(5)},X_{(49)}^{(5)} \end{array}\right],$$

$$\left[\begin{array}{l} X_{(1)}^{(6)},X_{(2)}^{(6)},X_{(3)}^{(6)},\boxed{X_{(4)}^{(6)},}X_{(5)}^{(6)},X_{(6)}^{(6)},X_{(7)}^{(6)},X_{(8)}^{(6)},X_{(9)}^{(6)},X_{(10)}^{(6)},\boxed{X_{(11)}^{(6)},}X_{(12)}^{(6)},X_{(13)}^{(6)},X_{(14)}^{(6)},X_{(15)}^{(6)},X_{(16)}^{(6)},X_{(17)}^{(6)},\boxed{X_{(18)}^{(6)},} \\ X_{(19)}^{(6)},X_{(20)}^{(6)},X_{(21)}^{(6)},X_{(22)}^{(6)},X_{(23)}^{(6)},X_{(24)}^{(6)},\boxed{X_{(25)}^{(6)},}X_{(26)}^{(6)},X_{(27)}^{(6)},X_{(28)}^{(6)},X_{(29)}^{(6)},X_{(30)}^{(6)},X_{(31)}^{(6)},\boxed{X_{(32)}^{(6)},}X_{(33)}^{(6)},X_{(34)}^{(6)},X_{(35)}^{(6)},X_{(36)}^{(6)}, \\ X_{(37)}^{(6)},X_{(38)}^{(6)},\boxed{X_{(39)}^{(6)},}X_{(40)}^{(6)},X_{(41)}^{(6)},X_{(42)}^{(6)},X_{(43)}^{(6)},X_{(44)}^{(6)},X_{(45)}^{(6)},\boxed{X_{(46)}^{(6)},}X_{(47)}^{(6)},X_{(48)}^{(6)},X_{(49)}^{(6)} \end{array}\right]$$

and

$$\left[\begin{array}{l} X_{(1)}^{(7)},X_{(2)}^{(7)},X_{(3)}^{(7)},\boxed{X_{(4)}^{(7)},}X_{(5)}^{(7)},X_{(6)}^{(7)},X_{(7)}^{(7)},X_{(8)}^{(7)},X_{(9)}^{(7)},X_{(10)}^{(7)},\boxed{X_{(11)}^{(7)},}X_{(12)}^{(7)},X_{(13)}^{(7)},X_{(14)}^{(7)},X_{(15)}^{(7)},X_{(16)}^{(7)},X_{(17)}^{(7)},\boxed{X_{(18)}^{(7)},} \\ X_{(19)}^{(7)},X_{(20)}^{(7)},X_{(21)}^{(7)},X_{(22)}^{(7)},X_{(23)}^{(7)},X_{(24)}^{(7)},\boxed{X_{(25)}^{(7)},}X_{(26)}^{(7)},X_{(27)}^{(7)},X_{(28)}^{(7)},X_{(29)}^{(7)},X_{(30)}^{(7)},X_{(31)}^{(7)},\boxed{X_{(32)}^{(7)},}X_{(33)}^{(7)},X_{(34)}^{(7)},X_{(35)}^{(7)},X_{(36)}^{(7)}, \\ X_{(37)}^{(7)},X_{(38)}^{(7)},\boxed{X_{(39)}^{(7)},}X_{(40)}^{(7)},X_{(41)}^{(7)},X_{(42)}^{(7)},X_{(43)}^{(7)},X_{(44)}^{(7)},X_{(45)}^{(7)},\boxed{X_{(46)}^{(7)},}X_{(47)}^{(7)},X_{(48)}^{(7)},X_{(49)}^{(7)} \end{array}\right]$$

so we have 7 DURSS

$$X^{(1)}_{(4)}, X^{(1)}_{(11)}, X^{(1)}_{(18)}, X^{(1)}_{(25)}, X^{(1)}_{(32)}, X^{(1)}_{(39)}, X^{(1)}_{(46)},$$
$$X^{(2)}_{(4)}, X^{(2)}_{(11)}, X^{(2)}_{(18)}, X^{(2)}_{(25)}, X^{(2)}_{(32)}, X^{(2)}_{(39)}, X^{(2)}_{(46)},$$
$$X^{(3)}_{(4)}, X^{(3)}_{(11)}, X^{(3)}_{(18)}, X^{(3)}_{(25)}, X^{(3)}_{(32)}, X^{(3)}_{(39)}, X^{(3)}_{(46)},$$
$$X^{(4)}_{(4)}, X^{(4)}_{(11)}, X^{(4)}_{(18)}, X^{(4)}_{(25)}, X^{(4)}_{(32)}, X^{(4)}_{(39)}, X^{(4)}_{(46)},$$
$$X^{(5)}_{(4)}, X^{(5)}_{(11)}, X^{(5)}_{(18)}, X^{(5)}_{(25)}, X^{(5)}_{(32)}, X^{(5)}_{(39)}, X^{(5)}_{(46)},$$
$$X^{(6)}_{(4)}, X^{(6)}_{(11)}, X^{(6)}_{(18)}, X^{(6)}_{(25)}, X^{(6)}_{(32)}, X^{(6)}_{(39)}, X^{(6)}_{(46)},$$
$$X^{(7)}_{(4)}, X^{(7)}_{(11)}, X^{(7)}_{(18)}, X^{(7)}_{(25)}, X^{(7)}_{(32)}, X^{(7)}_{(39)}, X^{(7)}_{(46)},$$

We select the sample unit from 7 DURSS

$$X^{(1)}_{(4)}, X^{(1)}_{(11)}, X^{(1)}_{(18)}, \boxed{X^{(1)}_{(25)}}, X^{(1)}_{(32)}, X^{(1)}_{(39)}, X^{(1)}_{(46)},$$
$$X^{(2)}_{(4)}, X^{(2)}_{(11)}, X^{(2)}_{(18)}, \boxed{X^{(2)}_{(25)}}, X^{(2)}_{(32)}, X^{(2)}_{(39)}, X^{(2)}_{(46)},$$
$$X^{(3)}_{(4)}, X^{(3)}_{(11)}, X^{(3)}_{(18)}, \boxed{X^{(3)}_{(25)}}, X^{(3)}_{(32)}, X^{(3)}_{(39)}, X^{(3)}_{(46)},$$
$$X^{(4)}_{(4)}, X^{(4)}_{(11)}, X^{(4)}_{(18)}, \boxed{X^{(4)}_{(25)}}, X^{(4)}_{(32)}, X^{(4)}_{(39)}, X^{(4)}_{(46)},$$
$$X^{(5)}_{(4)}, X^{(5)}_{(11)}, X^{(5)}_{(18)}, \boxed{X^{(5)}_{(25)}}, X^{(5)}_{(32)}, X^{(5)}_{(39)}, X^{(5)}_{(46)},$$
$$X^{(6)}_{(4)}, X^{(6)}_{(11)}, X^{(6)}_{(18)}, \boxed{X^{(6)}_{(25)}}, X^{(6)}_{(32)}, X^{(6)}_{(39)}, X^{(6)}_{(46)},$$
$$X^{(7)}_{(4)}, X^{(7)}_{(11)}, X^{(7)}_{(18)}, \boxed{X^{(7)}_{(25)}}, X^{(7)}_{(32)}, X^{(7)}_{(39)}, X^{(7)}_{(46)},$$

Let $X^{(1)}_{(25)}, X^{(2)}_{(25)}, X^{(3)}_{(25)}, X^{(4)}_{(25)}, X^{(5)}_{(25)}, X^{(6)}_{(25)}, X^{(7)}_{(25)}$ is DURSS of size 7

VI. Stratified Unified Ranked Set Sampling)SDURSS(

The population of N units is divided into L non-overlapping sub-groups known as strata each stratum have $N_1, N_2, \cdots, N_L$ units, respectively, such that $N_1 + N_2 + \cdots + N_L = N$. The size of the h^{th} strata denotes by N_h for $h = 1, 2, \cdots, L$. Then the samples are drawn independently from each stratum, producing samples sizes denoted by $n_1, n_2, ..., n_L$, such that the total sample size is $n = \sum_{h=1}^{L} n_h$. If the DURSS technique is applied for each stratum then the whole procedure is called a SDURSS. Define $X^{k(h)}_{[i]j}$ be the SDURSS sampled unit of the i^{th} rank, the j^{th} cycle in the h^{th} stratum, where $i = 1, 2, \cdots, m$; $j = 1, 2, \cdots, r$; $k = 1, 2, \cdots, m$; and $h = 1, 2, \cdots, L$. The mean of selected units is used as a population mean estimator.

Example 6: Suppose that we have two strata, i.e. $L = 2$ and $h = 1,2$. Let (m, r) Assume that from the first stratum we select a sample of size $m \times r = 6 \times 2 = 12$ and from the second stratum we want a sample of size $m \times r = 6 \times 2 = 12$ Then the process as illustrates as follows :

Stratum 1: *Now, select 12 samples as follows*:

Consider the case of $(\text{stratum1}, \ m = 6, \ r = 1)$ *and* $(\text{stratum1}, \ m = 6, \ r = 2)$. *Draw a simple random sample of size* $m^3 = 6^3 = 216$ *elements)6 sets of size 36 each(. Assume the elements are*

Stratum 1	$(r=1)$	$X^{1(1)}_{[1]1}, X^{1(1)}_{[2]1}, X^{1(1)}_{[3]1},\cdots, X^{1(1)}_{[36]1}, X^{2(1)}_{[1]1}, X^{2(1)}_{[2]1}, X^{2(1)}_{[3]1},\cdots, X^{2(1)}_{[36]1}, X^{3(1)}_{[1]1}, X^{3(1)}_{[2]1}, X^{3(1)}_{[3]1},\cdots, X^{3(1)}_{[36]1},$ $X^{4(1)}_{[1]1}, X^{4(1)}_{[2]1}, X^{4(1)}_{[3]1},\cdots, X^{4(1)}_{[36]1}, X^{5(1)}_{[1]1}, X^{5(1)}_{[2]1}, X^{5(1)}_{[3]1},\cdots, X^{5(1)}_{[36]1}, X^{6(1)}_{[1]1}, X^{6(1)}_{[2]1}, X^{6(1)}_{[3]1},\cdots, X^{6(1)}_{[36]1},$
	$(r=2)$	$X^{1(1)}_{[1]2}, X^{1(1)}_{[2]2}, X^{1(1)}_{[3]2},\cdots, X^{1(1)}_{[36]2}, X^{2(1)}_{[1]2}, X^{2(1)}_{[2]2}, X^{2(1)}_{[3]2},\cdots, X^{2(1)}_{[36]2}, X^{3(1)}_{[1]2}, X^{3(1)}_{[2]2}, X^{3(1)}_{[3]2},\cdots, X^{3(1)}_{[36]2},$ $X^{4(1)}_{[1]2}, X^{4(1)}_{[2]2}, X^{4(1)}_{[3]2},\cdots, X^{4(1)}_{[36]2}, X^{5(1)}_{[1]2}, X^{5(1)}_{[2]2}, X^{5(1)}_{[3]2},\cdots, X^{5(1)}_{[36]2}, X^{6(1)}_{[1]2}, X^{6(1)}_{[2]2}, X^{6(1)}_{[3]2},\cdots, X^{6(1)}_{[36]2},$

For $h=1$ we have: $X^{15(1)}_{[1]1}, X^{15(1)}_{[2]1}, X^{15(1)}_{[3]1}, X^{22(1)}_{[4]1}, X^{22(1)}_{[5]1}, X^{22(1)}_{[6]1}, X^{15(1)}_{[1]2}, X^{15(1)}_{[2]2}, X^{15(1)}_{[3]2}, X^{22(1)}_{[4]2}, X^{22(1)}_{[5]2}, X^{22(1)}_{[6]2}$

Stratum 2: *Now, select 12 samples as follows*:

Consider the case of $(\text{stratum2},\ m=6,\ r=1)$ *and* $(\text{stratum2},\ m=6,\ r=2)$. *Draw a simple random sample of size* $m^3=6^3=216$ *elements)6 sets of size 36 each(. Assume the elements are*

Stratum 2	$(r=1)$	$X^{1(2)}_{[1]1}, X^{1(2)}_{[2]1}, X^{1(2)}_{[3]1},\cdots, X^{1(2)}_{[36]1}, X^{2(2)}_{[1]1}, X^{2(2)}_{[2]1}, X^{2(2)}_{[3]1},\cdots, X^{2(2)}_{[36]1}, X^{3(2)}_{[1]1}, X^{3(2)}_{[2]1}, X^{3(2)}_{[3]1},\cdots, X^{3(2)}_{[36]1},$ $X^{4(2)}_{[1]1}, X^{4(2)}_{[2]1}, X^{4(2)}_{[3]1},\cdots, X^{4(2)}_{[36]1}, X^{5(2)}_{[1]1}, X^{5(2)}_{[2]1}, X^{5(2)}_{[3]1},\cdots, X^{5(2)}_{[36]1}, X^{6(2)}_{[1]1}, X^{6(2)}_{[2]1}, X^{6(2)}_{[3]1},\cdots, X^{6(2)}_{[36]1},$
	$(r=2)$	$X^{1(2)}_{[1]2}, X^{1(2)}_{[2]2}, X^{1(2)}_{[3]2},\cdots, X^{1(2)}_{[36]2}, X^{2(2)}_{[1]2}, X^{2(2)}_{[2]2}, X^{2(2)}_{[3]2},\cdots, X^{2(2)}_{[36]2}, X^{3(2)}_{[1]2}, X^{3(2)}_{[2]2}, X^{3(2)}_{[3]2},\cdots, X^{3(2)}_{[36]2},$ $X^{4(2)}_{[1]2}, X^{4(2)}_{[2]2}, X^{4(2)}_{[3]2},\cdots, X^{4(2)}_{[36]2}, X^{5(2)}_{[1]2}, X^{5(2)}_{[2]2}, X^{5(2)}_{[3]2},\cdots, X^{5(2)}_{[36]2}, X^{6(2)}_{[1]2}, X^{6(2)}_{[2]2}, X^{6(2)}_{[3]2},\cdots, X^{6(2)}_{[36]2},$

For $h=2$ we have: $X^{15(2)}_{[1]1}, X^{15(2)}_{[2]1}, X^{15(2)}_{[3]1}, X^{22(2)}_{[4]1}, X^{22(2)}_{[5]1}, X^{22(2)}_{[6]1}, X^{15(2)}_{[1]2}, X^{15(2)}_{[2]2}, X^{15(2)}_{[3]2}, X^{22(2)}_{[4]2}, X^{22(2)}_{[5]2}, X^{22(2)}_{[6]2}$

)Define: $X^{k(h)}_{[i]j}$, $k=$number of ranking the elements of each set, $h=$ stratum size, $i=$number ofeach set,
$r=$ number of cycles)times((

Therefore, the measured SDURSS units are

$X^{15(1)}_{[1]1}, X^{15(1)}_{[2]1}, X^{15(1)}_{[3]1}, X^{22(1)}_{[4]1}, X^{22(1)}_{[5]1}, X^{22(1)}_{[6]1}, X^{15(1)}_{[1]2}, X^{15(1)}_{[2]2}, X^{15(1)}_{[3]2}, X^{22(1)}_{[4]2}, X^{22(1)}_{[5]2}, X^{22(1)}_{[6]2}$

$X^{15(2)}_{[1]1}, X^{15(2)}_{[2]1}, X^{15(2)}_{[3]1}, X^{22(2)}_{[4]1}, X^{22(2)}_{[5]1}, X^{22(2)}_{[6]1}, X^{15(2)}_{[1]2}, X^{15(2)}_{[2]2}, X^{15(2)}_{[3]2}, X^{22(2)}_{[4]2}, X^{22(2)}_{[5]2}, X^{22(2)}_{[6]2}$

where their mean of these units is used as an estimator of the population mean.

To compare the efficiency of the empirical mean estimator based on SDURSS with their counterparts in SRS, SSRS, SRSS, and SMRSS via a simulation in R)Version 4.3.2(under the population of 100,000 units divided into two strata each stratum has 50,000 units with the numbers of set in each stratum $m=2,4,6,10$ and the number of cycles $r=2,5$. Using 5000 replications, estimates of the means, variances and mean square errors.

III. Results and Discussions

I. Estimation of Population Mean

Let $X_1, X_2,..., X_n$ be n independent random variables from a probability density function $f(x)$, with mean (μ) and variance (σ^2). The empirical mean estimator of DURSS is given by

$$\bar{X}_{DURSS}=\frac{1}{mr}\sum_{i=1}^{m}\sum_{j=1}^{r}X_{\left[l+(i-1)m\right]j} \tag{1}$$

where $l=\dfrac{m}{2}$ if i is an even number and $l=\dfrac{m+1}{2}$ if i is an odd number)for $i=1,2,...,m$(.

The DURSS variance can be estimated by

$$S^2_{DURSS} = \frac{1}{mr-1}\left\{\sum_{i=1}^{m}\sum_{j=1}^{r}\left(X_{[l+(i-1)m]j} - \bar{X}_{DURSS}\right)^2\right\}. \quad (2)$$

The SDURSS estimator of the population mean is given by

$$\bar{X}_{SDURSS} = \sum_{h=1}^{L} W_h\left(\bar{X}^h_{DURSS}\right) \quad (3)$$

Where $W_h = \frac{N_h}{N}$ and $\bar{X}^h_{FRSS}$ is the DURSS mean estimator in the h^{th} stratum.

The variance of $\bar{X}_{SFRSS}$ is given by

$$\begin{aligned} Var\left(\bar{X}_{SDURSS}\right) &= Var\left[\sum_{h=1}^{L}\frac{W_h}{m_h r}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r} X_{[l+(i-1)m_h]j}\right)\right] \\ &= \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r^2}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r} Var\left(X_{[l+(i-1)m_h]j}\right)\right) \\ &= \sum_{h=1}^{L}\frac{W_h^2}{m_h^2 r^2}\left(\sum_{i=1}^{m_h}\sum_{j=1}^{r} \sigma^2_{[l+(i-1)m_h]j,h}\right) \\ &= \sum_{h=1}^{L}\frac{W_h^2}{m_h r}\sigma^2_{[l+(i-1)m_h]j,h} \end{aligned} \quad)4($$

IV. Simulation Study

The simulation study is designed to investigate the performance of SDURSS for estimating the population mean compared to their counterparts in SRS, SSRS, and SRSS under parent asymmetric distributions: Exp)1(, Geo)0.5(, Gamma)0.5,1(, Gamma)1,2(, Beta)3,3(, Beta)9,2(, Weibull)0.5,1(, Weibull)1,2(, Log N)0,1(, Logistic)0,1(, CHI)1(. The simulations are done based on the population of 100,000 units is divided into two strata each stratum has 50,000 units, which are conducted for the numbers of set in each stratum $m = 2,4,6,10$ and the number of cycles $r = 2,5$ on 5,000 replications. If the underlying distribution is asymmetric, the efficiencies of SDURSS relative to SRS, SSRS, SRSS, and SMRSS, respectively are given by

$$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SRS}\right) = \frac{MSE\left(\bar{X}_{SRS}\right)}{MSE\left(\bar{X}_{SDURSS}\right)},$$

$$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SSRS}\right) = \frac{MSE\left(\bar{X}_{SSRS}\right)}{MSE\left(\bar{X}_{SDURSS}\right)},$$

$$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SRSS}\right) = \frac{MSE\left(\bar{X}_{SRSS}\right)}{MSE\left(\bar{X}_{SDURSS}\right)},$$

$$eff\left(\bar{X}_{SDURSS}, \bar{X}_{SMRSS}\right) = \frac{MSE\left(\bar{X}_{SMRSS}\right)}{MSE\left(\bar{X}_{SDURSS}\right)},$$

where MSE is the mean square error)MSE(.

The simulation results are shown in Tables 1-3.

Table 1: *The efficiency of SDURSS relative to SRS, SSRS, SRSS and SMRSS for estimating the population mean with* $m = 2$ *and* $r = 2,5$

Distribution	r	Efficiency			
		SDURSS vs. SRS	SDURSS vs. SSRS	SDURSS vs. SRSS	SDURSS vs. SMRSS
Exp)1(	2	0.7259	0.3542	0.6500	0.6256
	5	0.0409	0.0206	0.6197	0.6194
Geo)0.5(	2	0.7141	0.3609	0.6721	0.6717
	5	0.0426	0.0209	0.6467	0.6479
Gamma)0.5,1(	2	0.6433	0.3343	0.6179	0.6040
	5	0.0383	0.0195	0.6139	0.6088
Gamma)1,2(	2	0.7026	0.3572	0.6709	0.6739
	5	0.0401	0.0201	0.6086	0.6171
Beta)3,3(	2	0.8158	0.4088	0.7422	0.7366
	5	0.0443	0.0219	0.6806	0.6852
Beta)9,2(	2	0.7868	0.3959	0.7208	0.7107
	5	0.0429	0.0213	0.6623	0.6632
Weibull)0.5,1(	2	0.5839	0.2680	0.5658	0.5407
	5	0.0374	0.0186	0.5836	0.5369
Weibull)1,2(	2	0.7080	0.3512	0.6523	0.6644
	5	0.0404	0.0201	0.6165	0.6199
Log N)0,1(	2	0.5840	0.3203	0.5488	0.6177
	5	0.0366	0.0178	0.6153	0.5559
Logistic)0,1(	2	0.7376	0.3687	0.6582	0.6721
	5	0.0399	0.0201	0.6191	0.6138
CHI)1(	2	0.6550	0.3469	0.6387	0.6079
	5	0.0393	0.0200	0.6222	0.6233

Based on Table 1, the numbers of set in each stratum $m = 2$ and numbers of cycle $r = 2$, it indicates that the SDURSS estimator is less efficient than SRS, SSRS, SRSS and SMRSS estimators all asymmetric distributions.

Table 2: *The efficiency of SDURSS relative to SRS, SSRS, SRSS and SMRSS for estimating the population mean with* $m = 4$ *and* $r = 2,5$

Distribution	r	Efficiency			
		SDURSS vs. SRS	SDURSS vs. SSRS	SDURSS vs. SRSS	SDURSS vs. SMRSS
Exp)1(	2	0.9332	0.4654	0.4024	0.3737
	5	0.0617	0.0313	0.4720	0.1862
Geo)0.5(	2	0.9068	0.4551	0.3939	0.3768
	5	0.0618	0.0312	0.4675	0.2088
Gamma)0.5,1(	2	0.8045	0.3935	0.3509	0.3304
	5	0.0551	0.0278	0.4217	0.1454
Gamma)1,2(	2	0.9144	0.4566	0.3913	0.3603
	5	0.0611	0.0307	0.4724	0.1862
Beta)3,3(	2	**1.4000**	0.6931	0.5862	0.5552
	5	0.0808	0.0406	0.6180	0.3185

Distribution	r	Efficiency			
		SDURSS vs. SRS	SDURSS vs. SSRS	SDURSS vs. SRSS	SDURSS vs. SMRSS
Beta)9,2(	2	**1.2778**	0.6481	0.5556	0.5185
	5	0.0730	0.0365	0.5587	0.2628
Weibull)0.5,1(	2	0.5858	0.3235	0.2795	0.2676
	5	0.0440	0.0218	0.3354	0.0640
Weibull)1,2(	2	0.9209	0.4507	0.4038	0.3824
	5	0.0610	0.0303	0.4637	0.1839
Log N)0,1(	2	0.6315	0.2918	0.2912	0.2554
	5	0.0442	0.0237	0.3748	0.0823
Logistic)0,1(	2	**1.1039**	0.5582	0.4666	0.4395
	5	0.0649	0.0323	0.5025	0.1992
CHI)1(	2	0.7453	0.3833	0.3540	0.3336
	5	0.0543	0.0273	0.4172	0.1449

Based on Table 2, the numbers of set in each stratum $m=4$, we can conclude that the SDURSS estimator is less efficient compared to SRS, SSRS, SRSS and SMRSS estimators for the numbers of cycle $r=2$ underlying all asymmetric distributions.

Table 3: *The efficiency of SDURSS relative to SRS, SSRS, SRSS and SMRSS for estimating the population mean with* $m=6$ *and* $r=2,5$

Distribution	r	Efficiency			
		SDURSS vs. SRS	SDURSS vs. SSRS	SDURSS vs. SRSS	SDURSS vs. SMRSS
Exp)1(	2	0.1401	0.7593	0.4259	0.2994
	5	0.0856	0.0427	0.4305	0.1173
Geo)0.5(	2	**1.4798**	0.7489	0.4205	0.4103
	5	0.0868	0.0430	0.4352	0.1515
Gamma)0.5,1(	2	**1.2423**	0.6121	0.3599	0.1710
	5	0.0730	0.0362	0.3663	0.0909
Gamma)1,2(	2	**1.5048**	0.7504	0.4287	0.1875
	5	0.0848	0.0424	0.4296	0.1164
Beta)3,3(	2	**2.4839**	**1.2516**	0.6903	0.2645
	5	0.1263	0.0630	0.6389	0.2194
Beta)9,2(	2	**2.0938**	**1.0469**	0.5781	0.2344
	5	0.1102	0.0547	0.5534	0.1742
Weibull)0.5,1(	2	0.8754	0.4244	0.2435	0.0809
	5	0.0503	0.0245	0.2469	0.0344
Weibull)1,2(	2	**1.5581**	0.7616	0.4327	0.1887
	5	0.0849	0.0430	0.4331	0.1194
Log N)0,1(	2	0.8643	0.4264	0.2589	0.0808
	5	0.0557	0.0273	0.2816	0.0444
Logistic)0,1(	2	**1.8218**	0.9101	0.5095	0.1399
	5	0.0929	0.0466	0.4730	0.1166
CHI)1(	2	**1.2097**	0.5977	0.3581	0.2171
	5	0.0723	0.0367	0.3665	0.0985

Based on Table 3, the numbers of set in each stratum $m=6$, it implies that the SDURSS estimator is less efficient than SRS, SSRS, SRSS and SMRSS estimators for the numbers of cycle $r=2$ based on all asymmetric distributions.

Table 4: The efficiency of SDURSS relative to SRS, SSRS, SRSS and SMRSS for estimating the population mean with $m=10$ and $r=2,5$

Distribution	r	Efficiency			
		SDURSS vs. SRS	SDURSS vs. SSRS	SDURSS vs. SRSS	SDURSS vs. SMRSS
Exp)1(	2	0.0860	0.0432	0.0139	0.0044
	5	0.0843	0.0417	0.2568	0.0522
Geo)0.5(	2	0.0864	0.0430	0.0143	0.0065
	5	0.0846	0.0417	0.2531	0.0799
Gamma)0.5,1(	2	0.0735	0.0371	0.0122	0.0043
	5	0.0717	0.0356	0.2148	0.0462
Gamma)1,2(	2	0.0863	0.0430	0.0142	0.0042
	5	0.0843	0.0414	0.2531	0.0513
Beta)3,3(	2	0.1287	0.0643	0.0205	0.0051
	5	0.1246	0.0623	0.3758	0.0780
Beta)9,2(	2	0.1110	0.0555	0.0179	0.0043
	5	0.1085	0.0538	0.3271	0.0649
Weibull)0.5,1(	2	0.0512	0.0255	0.0086	0.0023
	5	0.0488	0.0244	0.1510	0.0203
Weibull)1,2(	2	0.0877	0.0438	0.0143	0.0044
	5	0.0846	0.0425	0.2560	0.0523
Log N)0,1(	2	0.0540	0.0275	0.0092	0.0022
	5	0.0530	0.0280	0.1627	0.0223
Logistic)0,1(	2	0.0944	0.0473	0.0153	0.0026
	5	0.0919	0.0462	0.2773	0.0382
CHI)1(	2	0.0754	0.0369	0.0126	0.0058
	5	0.0710	0.0360	0.2185	0.0595

Based on Table 4, the numbers of set in each stratum $m=10$, it implies that the SDURSS estimator is less efficient than SRS, SSRS, SRSS and SMRSS estimators for the numbers of cycle $r=2$ based on all asymmetric distributions.

V. Real Data example

In this section, the application of the proposed sampling method is shown by using a real data example. The researcher went to the area to collect data by himself. The data sets used in this example include: There are a total of 5 plots of False pakchoi, with a length of 20 meters and a width of 1 meter. Each plant will have a minimum number of flowers of 3 flowers per plant. If data is collected in batches, it will be 25 plants per batch with 75-150 flowers. Where 1 plot can store 20 data sets from a total of 5 plots, totaling 100 data sets. Figure 1-2 illustrate False pakchoi and Table 5 represent Number set and real data.

Figure 1

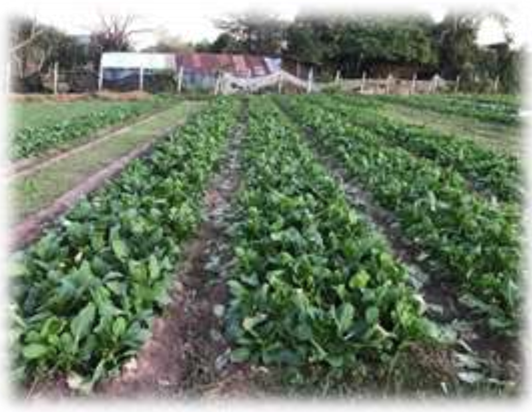

Figure 2

Table 5: *Number set and real data for False pakchoi*

Number set	data	Number set	data	Number set	data	Number set	data	Number set	data
Set 1	103	Set 21	125	Set 41	109	Set 61	89	Set 81	97
Set 2	115	Set 22	123	Set 42	141	Set 62	131	Set 82	148
Set 3	103	Set 23	129	Set 43	133	Set 63	123	Set 83	118
Set 4	117	Set 24	118	Set 44	114	Set 64	144	Set 84	90
Set 5	150	Set 25	99	Set 45	138	Set 65	128	Set 85	125
Set 6	110	Set 26	97	Set 46	111	Set 66	149	Set 86	104
Set 7	123	Set 27	92	Set 47	116	Set 67	123	Set 87	90
Set 8	102	Set 28	146	Set 48	146	Set 68	148	Set 88	129
Set 9	143	Set 29	143	Set 49	145	Set 69	105	Set 89	108
Set 10	76	Set 30	115	Set 50	132	Set 70	120	Set 90	110
Set 11	97	Set 31	76	Set 51	129	Set 71	114	Set 91	118
Set 12	135	Set 32	99	Set 52	127	Set 72	125	Set 92	118
Set 13	140	Set 33	117	Set 53	108	Set 73	130	Set 93	126
Set 14	81	Set 34	112	Set 54	107	Set 74	120	Set 94	83
Set 15	99	Set 35	89	Set 55	136	Set 75	137	Set 95	114
Set 16	136	Set 36	130	Set 56	97	Set 76	139	Set 96	121
Set 17	93	Set 37	111	Set 57	99	Set 77	84	Set 97	97
Set 18	103	Set 38	142	Set 58	112	Set 78	115	Set 98	108
Set 19	83	Set 39	136	Set 59	97	Set 79	147	Set 99	94
Set 20	90	Set 40	96	Set 60	123	Set 80	142	Set 100	113

Total Population flower False pakchoi is 11,636, population mean $\bar{X} = 116.36$ to collect a sample of size 8, using set size is $m = 4$ and number of cycles)times(is $r = 2$ in SRS, SSRS, SRSS, and SDURSS designs, DURSS technique can be described as follows:

I. Draw a simple random sample of size $m^3 = 4^3 = 64$ elements)4 sets of size 16 each(.

II. Use the usual URSS procedure on each set to obtain m ranked set samples of size m each.

III. Apply the URSS procedure again on step)2(to obtain a DURSS of size 8.

The measured values in both SRS, SSRS, SRSS, SMRSS, and SDURSS and designs are presented in Table 6.

Table 6: *Sampled units in SRS, SSRS, SRSS, SMRSS, and SDURSS designs.*

	SRS	117	90	97	123	146	145	108	94
SSRS	Stratum 1	115	104	92	132	114	140	114	99
	Stratum 2	111	146	148	97	118	107	84	97
SRSS	Stratum 1	90	99	109	144	76	90	105	129
	Stratum 2	90	114	143	145	81	97	111	120
SMRSS	Stratum 1	76	108	142	148	81	96	114	115
	Stratum 2	97	111	103	144	89	94	104	127
SDURSS	Stratum 1	123	129	146	150	81	83	112	136
	Stratum 2	104	109	125	148	83	97	102	139

$\bar{X}_{SRS} = 115$

$\bar{X}_{SSRS(stratum1)} = 113.75, \bar{X}_{SSRS(stratum2)} = 113.5$

$\bar{X}_{SRSS(stratum1)} = 105.25, \bar{X}_{SRSS(stratum2)} = 112.63$

$\bar{X}_{SMRSS(stratum1)} = 110, \bar{X}_{SMRSS(stratum2)} = 108.63$

$\bar{X}_{SDURSS(stratum1)} = 120, \bar{X}_{SDURSS(stratum2)} = 113.38$

$S^2_{SRS} = 481.15$

$S^2_{SSRS} = 20.7731$

$S^2_{SRSS} = 29.2044$

$S^2_{SMRSS} = 41.1914$

$S^2_{SDURSS} = 37.23332$

VI. Conclusion

In conclusion, the proposed estimator in SDURSS provide efficient their counterparts in SRS, SSRS, SRSS, and SMRSS for eleven parent asymmetric distributions in the case of a larger sample size and small size numbers of cycle. For the small sample size, the proposed estimator in SDURSS still provide less efficient than four methods, but it gives more efficient than SRS and SSRS in some cases.

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