RT&A, No 1 (82)
Volume 20, March 2025

OPTIMIZING BAYESIAN REPETITIVE GROUP SAMPLING PLAN FOR QUALITY CONTROL TO ENHANCE DECISION MAKING EFFICIENCY IN MODERN MANUFACTURING

P. Sivakumar[1*], V. Kaviyarasu[2] and V. Devika[3]

•

[*1]Assistant Professor, Department of Statistics, St. Joseph's University, Bangalore.
[2]Associate Professor, Department of Statistics, Bharathiar University, Coimbatore.
[3]Assistant Professor, Department of Mathematics, Amrita Vishwa Vidyapeetham University, Coimbatore.
[*1]sivabustat936@gmail.com, [2]kaviyarasu@buc.edu.in and [3]devumol20@gmail.com

Abstract

This article introduces an approach to optimize the design of Repetitive Group Sampling (RGS) plan in the context of quality control for modern manufacturing processes. The primary objective of this study is to enhance decision-making efficiency by applying Bayesian principles to develop optimal sampling plans. In modern manufacturing environment, the industries are using the advanced technologies and machineries to maintain the quality of their products. The existence of defects would consequently be highly rare in such production. In such situation, Zero Inflated Poisson (ZIP) distribution is a more appropriate probability distribution rather than the usual Poisson distribution. Further, manufacturing industries often use a variety of manpower and materials to produce their products in various production streams. This may lead to have more quality variation in between lots and hence, the lot quality will vary over lots. The lots that arise from such a production process will be unstable, and quality variations among the units are often heterogeneous in nature. In such situation, the Bayesian sampling plans under Zero Inflated Poisson distribution would be more effective and alternative for traditional sampling plans. This paper studies the designing and selection of Bayesian Repetitive Group Sampling (BRGS) Plan under the conditions of Gamma-Zero Inflated Poisson distribution (G-ZIP). To investigate the effectiveness of this plan, a comparison between the proposed BRGS plan and various existing sample plans is made. Further, we provided the procedure and tables with the suitable numerical illustration to compute the optimal sampling plan.

Keywords: Bayesian perspective, group sampling, quality control, decision-making, process monitoring.

1. INTRODUCTION

Acceptance sampling plans are useful tools for applications involving quality assurance and provide the producer and the consumer a general rule for lot sentencing to satisfy their requirements for product quality. A well-designed acceptance sampling plan not only reduces inspection costs and time, but also protects both the producer and the consumer. Therefore, a sampling plan with a smaller sample size required for inspection is more desirable and useful,

particularly when inspection is costly and destructive. There are several sampling schemes that have been used for various situations in the industry. One of the sampling schemes called the repetitive sampling plan has been introduced by Sherman. This sampling procedure is especially effective for situations where product inspection is destructive and costly. It enables the acceptance or rejection of a lot based on repeated testing of a small number of identical samples. The operating procedure of the attributes RGS plan can be seen in Sherman [1]. Further, the RGS plans are usually more efficient than the single sampling plan. The RGS plan can achieve the desired level of protection with a smaller sample size compared to a single sampling plan. Soundararajan and Ramasamy [2] developed procedures and tables to select the optimal parameters for the RGS plan. Balamurali et al., [3] developed the RGS plan and demonstrated that it significantly reduces the average sample size compared to single and double sampling plans. Recently, Kannan et.al., [4] proposed economic designed RGS plan to satisfy both producer's and consumer's risks with minimum cost under Birnbaum–Saunders distribution. However, recent attempts to develop procedures for Repetitive Group Sampling (RGS) plans specifically for attribute quality characteristics have been limited. Hence, this paper attempts to extend the concept of repetitive group sampling based on attribute quality characteristics.

In recent years, due to technological advancements there is a competitive environment in manufacturing industries has produce products in perfect quality. However, in production processes, there may be natural variability that causes some batches to have zero defects while others have a Poisson-distributed number of defects. ZIP distributions allow for the modeling of this variability effectively. Incorporating the ZIP distribution into acceptance sampling plans can lead to more efficient and cost-effective sampling plans. By accurately modeling the likelihood of zero defects, we can design acceptance sampling plans that reduce the number of samples required for inspection when the process is consistently producing zero-defective items. ZIP distribution is appropriate to design sampling plans that are more conservative when it comes to rejecting batches, reducing the risk of rejecting batches with only a small number of defects or no defects at all. Further, the application of ZIP model to defects in the manufacturing process has been discussed by Lambert [5]. In dental epidemiology research, Bohning et al., [6] made a few significant comparisons between the ZIP distribution and the Poisson distribution to measure people's dental health. Some of the applications of ZIP distribution can be found in Xie et al., [7], Rodrigues [8], Naya et al., [9], Sim and Lim [10], Yang et al., [11], Mussida et al., [12]. Further, Loganathan and Shalini [13] developed a single sampling plan based on attributes under the ZIP distribution. Additionally, Rao [14] designed a single sampling plan for resubmitted lots considering the ZIP distribution.

In Bayesian methodology, the acceptance sampling plans includes prior information regarding the process variations for taking decisions about the submitted lots can be used under the sampling plans as an alternative to the traditional sampling plans. The optimal design of repetitive group sampling plans under a Bayesian perspective represents a significant advancement in quality control for modern manufacturing. It enables enhanced decision-making efficiency, leading to improved process performance, reduced costs, and increased customer satisfaction. The proposed methodology has practical implications for manufacturers seeking to optimize their quality control processes and navigate the challenges of the dynamic manufacturing landscape. Bayesian methods offer a flexible framework for modeling and estimation, allowing for more accurate and reliable decision-making. Several studies have successfully applied Bayesian techniques in various quality control applications, including process monitoring, parameter estimation, and defect prediction. Hald [15] provided a detailed discussion on the procedures and implications of a Bayesian Single

Sampling Plan, considering both Gamma-Poisson (GP) and Beta-Binomial distributions.

Additionally, Calvin [16], Case et al., [17], and Guthrie and Johns [18] explored the selection of prior distributions for the lot fraction of non-conforming items in Bayesian sampling plans. Vijayaraghavan et al., [19] discussed a Bayesian Single Sampling Plan using the Gamma-Poisson distribution and proposed a method to evaluate the efficiency of this sampling plan in comparison to the conventional Poisson Single Sampling Plan. Rajagopal et al., [20] developed a Bayesian Single Sampling Plan based on the Polya distribution, utilizing the Guenther approach, and examined the discriminating power of these plans through their corresponding operating characteristic curves.

The purpose of this article is to develop a Bayesian Repetitive Group Sampling (BRGS) plan by attribute, based on the G-ZIP distribution, using the Guenther approach. In next section, provides an operating procedure of Repetitive Group Sampling plan. In third section, a brief description of the Bayesian G-ZIP distribution along with its performance measure is given. The fourth section provides the operating procedure and tables for selecting the proposed plan and to determine the optimum plan parameters for the specified quality levels through the Guenther iterative producer. In the fifth section, numerical illustrations are provided to assess the effectiveness of proposed plan compared to alternative sampling plans. The results are summarized in the concluding section.

2. REPETITIVE GROUP SAMPLING PLAN

Sherman [1] introduced the Repetitive Group Sampling (RGS) plan, which provides a straightforward procedure for attribute-based quality characteristics. This plan is more efficient than single sampling and resembles sequential sampling methods. It is particularly suited for situations involving destructive or costly inspections, using repeated samples to determine lot acceptance or rejection. The RGS plan achieves the desired protection with a smaller sample size compared to single sampling. According to Sherman [1] the operating procedure of RGS plan involves the following steps. First, a random sample of size n is taken from the lot. Next, the number of non-conforming items, d in the sample is observed. If $d \leq c_1$, the lot is accepted. If $d > c_2$, the lot is rejected. If $c_1 < d \leq c_2$, the sampling and inspection process is repeated until a decision is made. This plan is completely specified by three parameters, namely sample size n and the acceptance numbers c_1 and c_2. It is observed that the RGS plan reduces to a single sampling plan when $c_1 = c_2$ with c_1 always being less than c_2.
The probability of lot acceptance is determined by using the Operating Characteristic (OC) function, which is derived to be:

$$P_A(p) = \frac{P_a(p)}{P_a(p)+P_r(p)} \quad (1)$$

Where, $P_a(p)$ is the probability of acceptance of a submitted lot with fraction defective p based on a given sample, whereas $P_r(p)$ is the corresponding probability of lot rejection.

3. THE OC FUNCTION OF BAYESIAN RGS PLAN UNDER G-ZIP DISTRIBUTION

In acceptance sampling plan the number of nonconforming occurrences during a sampling inspection is considered as count data, their Poisson frequency distribution model could be identified using a probability distribution with parameter, representing the average number of defects per unit. However, the Zero-Inflated Poisson count models provide an alternative method to explain the excess zeros that is a greater number of non-conforming units by modeling the data

as a mixture of two separate distributions.

As given by Lambert [5] the probability function of a Zero Inflated Poisson distribution is given by,

$$P(X = x; \omega, \lambda) = \begin{cases} \omega + (1-\omega)e^{-\lambda}, & when\ x = 0 \\ (1-\omega)\frac{e^{-\lambda}\lambda^{x}}{x!}, & when\ x = 1,2,3 \ldots \end{cases} \quad (2)$$

Where, ω and λ are the parameters of ZIP distribution with $0 < \omega < 1,\ \lambda > 0$. When $\omega = 0$, this model reduces to the Poisson model. Further, when the manufacturing process is well monitored, defects become a rare event, resulting in many sampled products having zero defects. In such cases, the ZIP distribution is the appropriate probability distribution for modeling the number of defects in the sampled products. When the number of nonconformities items in the sample is followed by the model of Zero Inflated Poisson distribution with parameter (np, ω), when the proportion of nonconformities p varies at random from lot by lot and is distributed according to a gamma distribution, which is a natural conjugate prior to p, the density function of the p is given by,

$$f(p/t, s) = \frac{e^{-tp}t^{s}p^{s-1}}{\Gamma s}, \qquad 0 \le p < \infty, \qquad t, s > 0$$

Where, t is scale parameter and s is the shape parameter. If $E(p) = \bar{p}$ is gives the scale parameter is obtained by $t = s/\bar{p}$. Here, the prior knowledge s is estimated from past history of the production process. Further, the Uniform distribution is assumed to be the conjugate prior to ω with parameters a and b. The probability density function of the ω is defined as,

$$f(\omega/a, b) = \frac{1}{b-a}\ , a \le \omega \le b$$

In particular, the limitation of parameter ω can be taking $a = 0$ and $b = 1$, that is the uniform prior on (0,1). Then the equation for the standard uniform distribution is,

$$f(\omega) = 1 \quad \text{for} \quad 0 \le \omega \le 1$$

Thus, the predictive distribution of the number of defectives x is reduced to the G-ZIP distribution. In cases where the production process is unstable, the non-conforming items x and the average number of defects p are independently distributed. According to Hald [15] the average probability of acceptance $\bar{p}$ is approximately obtained by,

$$P_a(\bar{p}) = \int_0^{\infty} P_a(p)\, f(p) dp \quad (3)$$

Thus, the average probability of acceptance $\bar{p}$ under the conditions of Gamma prior distribution and ZIP sampling distribution can be obtained by,

$$p(x; \omega, n\bar{p}, s) = \begin{cases} \omega + (1-\omega)(1-\rho)^{s}, & when\ x = 0 \\ (1-\omega)\binom{x+s-1}{s-1}\rho^{x}(1-\rho)^{s}, & when\ x = 1,2,3 \ldots \end{cases} \quad (4)$$

Let us take for convenience $\rho = \left(\frac{n\bar{p}}{n\bar{p}+s}\right)$. It is to be observed that the sampling distribution of x is the Zero Inflated Negative binomial model with parameter s and $\left(\frac{n\bar{p}}{n\bar{p}+s}\right)$ When production is not stable, both x and $\bar{p}$ are independently distributed, and hence the sampling distribution of x, according to

Hald [15] under the conditions that $\bar{p} < 0.1$, $\bar{p}/s < 0.2$ the OC function is given by,

$$P_a(\bar{p}) = \sum_{x=0}^{c} p(x; \omega, n\bar{p}, s) \tag{5}$$

where $\bar{p}$ is the average lot quality or average fraction non-conforming. Here, the value of s can be estimated from the prior information about the production process. When $c = 0$, the lot acceptance probability becomes as,

$$P_a(\bar{p}) = \omega + (1-\omega)(1-\rho)^s \tag{6}$$

Based on this, the OC function of Bayesian RGS plan under the conditions of Gamma-Zero Inflated Poisson distribution is given by,

$$P_A(\bar{p}) = \frac{\omega + (1-\omega)(1-\rho)^s + \sum_{x=1}^{c_1} p(x; \omega, n\bar{p}, s)}{1 - \sum_{x=1}^{c_2} p(x; \omega, n\bar{p}, s) + \sum_{x=1}^{c_1} p(x; \omega, n\bar{p}, s)}$$

4. DETERMINATION OF PLAN PARAMETER

The optimal sampling plans are determined for a fixed parameters ω, s and a wide range of p_1 and p_2 with specified producer's and consumer's risks which are presented in Table 5.1- 5.8. The plan parameters are obtained from these tables with producer's risk to a maximum of 5% and consumer's risk to a maximum of 10%. Under these conditions, can be determine the plan parameters and minimize the Average Sample Number (ASN) at the level of Limiting Quality Level (LQL). The ASN of the RGS plan is given by,

$$ASN = \frac{n}{P_a(p) + P_r(p)} \tag{7}$$

Where $P_a(p)$ and $P_r(p)$ are the probability of acceptance and probability of rejection of a lot, under G-ZIP model. It should be noted that, the values of the parameter s, in the prior distribution range over the interval $(0, \infty)$. It is observed from the empirical study that the values of $P_a(p)$ of BRGS plan by attributes under the conditions of G-ZIP distribution do not differ much from those of ZIP plans at each value of p for larger values of s. It indicates that the OC function under the conditions of G-ZIP converges to non-Bayesian ZIP sampling plans. Hence, two different values such as 5 and 10 are taken for s. The set of tables corresponds to these values of s with the fixed parameter ω = 0.001,0.01,0.05,0.09 are considered. The BRGS plan can be used for the situation where the shape parameter is known and unknown. Normally, producers keep the record of the estimated shape parameter value for their product or it can be estimated from the available data. While searching for the optimum sampling plan using the iterative procedure [22], the values of n and c were restricted to the maximum of 7500 and 75 respectively for the iteration purpose. In some cases, very large values were obtained for n, which are not feasible to apply in practice. To those data sets, the optimum sampling plans are not presented which are denoted by the symbol ***.

5. NUMERICAL ILLUSTRATION

In this section, the procedure of selecting the plan parameters for the proposed plan is described with numerical illustrations. The significance of the Bayesian RGS plan under the conditions of G-ZIP distribution is highlighted.

Table.5.1 *Optimal BRGS plan under G-ZIP distribution for given* $p_1, p_2, \alpha = 0.05, \beta = 0.10, s = 5$ *and* $\omega = 0.001$.

AQL (p_1)	**LQL** (p_2)					
	0.05	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
0.005	72; 0,2 (104.85)	54; 0,1 (64.21)	47;0,1 (55.833)	41;0,1 (48.745)	36; 0,1 (42.937)	33;0,1 (39.171)
0.01	79; 0,3 (141.668)	66;0,3 (118.110)	51;0,2 (74.609)	45; 0,2 (65.533)	40; 0,1 (58.252)	36; 0,2 (52.426)
0.015	94; 0,5 (249.636)	72; 0,4 (157.949)	56;0,3 (101.052)	49;0,3 (88.422)	44;0,3 (78.740)	36;0,2 (52.427)
0.02	117;0,8 (516.976)	85;0,6 (268.894)	68;0,5 (177.547)	54;0,4 (118.462)	44;0,3 (78.740)	40;0,3 (70.996)
0.025	152;0,13 (1387.722)	103;0,9 (538.750)	78;0,7 (294.534)	59;0,5 (155.794)	48;0,4 (105.299)	44;0,4 (94.623
0.03	262;0,29 (9932.649)	127;0,13 (1150.453)	89;0,9 (457.218)	68;0,7 (258.482)	57;0,6 (178.655)	47;0,5 (124.818)
0.035	***	190;0,24 (5104.563)	109;0,13 (983.553)	82;0,10 (494.337)	65;0.8 (287.209)	51;0,6 (161.337)
0.04	***	***	153;0,22 (3515.175)	95;0,13 (867.326)	73;0,10 (438.453)	58;0,8 (260.776)
0.045	***	***	***	130;0,21 (2708.636)	89;0,14 (915.086)	66;0,10 (398.724)
0.05	***	***	***	***	112;0,20 (2116.08)	80;0,14 (809.744)

Table.5.2 *Optimal BRGS plan under G-ZIP distribution for given* $p_1, p_2, \alpha = 0.05, \beta = 0.10, s = 5$ *and* $\omega = 0.01$.

AQL (p_1)	**LQL** (p_2)					
	0.05	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
0.005	75;0,2 (106.448)	57;0,1 (66.937)	49;0,1 (57.500)	43;0,1 (50.423)	38;0,1 (44.625)	34;0,1 (39.986)
0.01	83;0,3 (142.062)	70;0,3 (118.685)	54;0,2 (76.324)	47;0,2 (66.617)	42;0,2 (59.363)	38;0,2 (53.562)
0.015	100;0,5 (241.745)	76;0,4 (155.790)	60;0,3 (101.730)	52;0,3 (88.833)	46;0,3 (78.885)	38;0,2 (53.562)
0.02	135;0,9 (564.108)	98;0,7 (317.435)	72;0,5 (172.095)	57;0,4 (116.843)	51;0,4 (103.854)	42;0,3 (71.211)
0.025	***	128;0,11 (653.018)	84;0,7; (272.087)	68;0,6 (191.242)	56;0,5 (133.852)	46;0,4 (93.438)
0.03	***	***	117;0,12 (642.718)	79;0,8 (291.620)	70;0,7 (201.816)	50;0,5 (120.738)
0.035	***	***	***	108;0,13 (646.546)	75;0,9 (313.394)	59;0,7 (189.999)
0.04	***	***	***	***	96;0,13 (574.708)	72;0,10 (335.409)
0.045	***	***	***	***	***	91;0,14 (588.234)
0.05	***	***	***	***	***	***

Table.5.3 *Optimal BRGS plan under G-ZIP distribution for given* $p_1, p_2, \alpha = 0.05, \beta = 0.10, s = 5$ *and* $\omega = 0.05$.

AQL (p_1)	**LQL** (p_2)					
	0.05	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
0.005	99;0,2 (122.031)	83;0,2 (102.068)	71;0,2 (87.379)	56;0,1 (61.64)	49;0,1 (54.100)	45;0,1 (49.497)
0.01	140;0,5 (224.654)	104;0,4 (155.587)	80;0,3 (108.986)	70;0,3 (95.363)	55;0,2 (69.794)	49;0,2 (60.641)
0.015	409;0,19 (811.944)	145;0,7 (254.984)	100;0,5 (140.467)	75;0,4 (115.645)	70;0,4 (103.974)	56;0,3 (76.290)
0.02	***	1015;0,58 (2027.701)	204;0,13 (400.903)	108;0,7 (191.332)	78;0,5 (124.846)	70;0,5 (112.327)
0.025	***	***	***	358;0,28 (758.619)	126;0,10 (241.031)	87;0,7 (152.957)
0.03	***	***	***	***	676;0,58 (1352.596)	215;0,20 (426.644)
0.035	***	***	***	***	***	***

Table.5.4 *Optimal BRGS plan under G-ZIP distribution for given* $p_1, p_2, \alpha = 0.05, \beta = 0.10, s = 5$ *and* $\omega = 0.09$.

AQL (p_1)	**LQL** (p_2)					
	0.05	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
0.005	239;0,4 (260.462)	170;0,3 (183.289)	153;0,3 (163.427)	11;0,2 (117.033)	100;0,2 (105.238)	89;0,2 (93.804)
0.01	***	886;0,25 (985.032)	360;0,11 (400.36)	191;0,6 (211.871)	152;0,5 (167.066)	119;0,4 (129.815)
0.015	***	***	***	1549;0,61 (1726.471)	591;0,25 (656.937)	291;0,13 (323.966)
0.02	***	***	***	***	***	***

Table.5.5 *Optimal BRGS plan under G-ZIP distribution for given* $p_1, p_2, \alpha = 0.05, \beta = 0.10, s = 10$ *and* $\omega = 0.001$.

AQL	LQL (p_2)					
(p_1)	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
0.005	57;0,1 (69.554)	48;0,1 (58.376)	41;0,1 (49.919)	36;0,1 (43.783)	32;0,1 (38.919)	29;0,1 (35.193)
0.01	71;0,3 (140.775)	53;0,3 (82.117)	46;0,2 (70.677)	40;0,2 (61.715)	36;0,2 (55.081)	29;0,1 (35.193)
0.015	78;0,4 (199.729)	59;0,3 (117.322)	51;0,4 (100.536)	40;0,2 (61.715)	36;0,2 (55.084)	32;0,2 (49.372)
0.02	93;0,6 (385.083)	71;0,5 (233.454	56;0,4 (142.347)	44;0,3 (88.007)	40;0,3 (78.176)	36;0,3 (70.358)
0.025	114;0,9 (945.404)	77;0,6 (323.137)	61;0,5 (199.759)	49;0,4 (124.580)	44;0,4 (110.305)	36;0,3 (70.358)
0.03	165;0,16 (4826.770)	96;0,9 (771.389)	71;0,7 (379.076)	58;0,6 (241.223)	48;0,5 (154.045)	39;0,4 (99.861)
0.035	171;0,21 (8081.907)	126;0,14 (2620.430)	82;0,9 (665.819)	63;0,7 (325.282)	52;0,6 (212.464)	43;0,5 (139.114)
0.04	***	***	102;0,13 (1846.700)	72;0,9 (578.504)	56;0,7 (289.139)	46.0,6 (194.784)
0.045	***	***	320;0,46 (31309.960)	90;0,13 (1573.900)	64;0,9 (514.226)	50;0,7 (263.145)
0.05	***	***	***	118;0,13 (4987.170)	76;0,12 (1108.500)	57;0,9 (472.702)

Table.5.6 *Optimal BRGS plan under G-ZIP distribution for given* $p_1, p_2, \alpha = 0.05, \beta = 0.10, s = 10$ *and* $\omega = 0.01$.

AQL	LQL (p_2)					
(p_1)	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
0.005	62;0,2 (99.826)	50;0,1 (59.932)	43;0,1 (51.489)	38;0,1 (45.365)	34;0,1 (40.509)	30;0,1 (35.959)
0.01	74;0,4 (139.493)	56;0,2 (83.276)	48;0,2 (71.380)	42;0,2 (62.457)	37;0,2 (55.344)	30;0,1 (35.959)
0.015	82;0,4 (193.514)	62;0,3 (116.243)	53;0,3 (99.637)	47;0,3 (87.182)	37;0,2 (55.344)	34;0,2 (50.175)
0.02	108;0,7 (458.610)	75;0,5 (220.322)	59;0,4 (137.877)	47;0,3 (87.182)	41;0,3 (77.496)	37;0,3 (69.746)
0.025	182;0,14 (1575.390)	90;0,7 (382.166)	71;0,6 (250.271)	57;0,5 (163.766)	46;0,4 (107.148)	37;0,3 (69.749)
0.03	***	152;0,14 (1304.828)	84;0,8 (415.720)	62;0,6 (219.427)	50;0,5 (146.881)	41;0,4 (96.756)
0.035	***	***	130;0,14 (1125.282)	66;0,7 (295.112)	55;0,6 (195.438)	45;0,5 (132.193)
0.04	***	***	***	171;0,21 (1653.542)	71;0,9 (399.379)	54;0,7 (229.300)
0.045	***	***	***	498;0,59 (4960.443)	101;0,14 (877.904)	64;0,9 (358.414)
0.05	***	***	***	***	365;0,49 (3679.178)	91;0,14 (787.621)

Table.5.7 *Optimal BRGS plan under G-ZIP distribution for given* $p_1, p_2, \alpha = 0.05, \beta = 0.10, s = 10$ *and* $\omega = 0.05$.

AQL	**LQL (p_2)**					
(p_1)	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
0.005	85;0,2 (108.512)	71;0,2 (90.530)	54;0,1 (60.372)	47;0,1 (52.613)	42;0,1 (46.956)	38;0,1 (42.431)
0.01	96;0,3 (136.436)	81;0,3 (116.526)	61;0,2 (77.687)	53;0,2 (67.741)	48;0,2 (60.773)	43;0,2 (54.569)
0.015	159;0,7 (300.280)	105;0,5 (179.863)	79;0,4 (125.816)	60;0,3 (87.147)	54;0,3 (77.684)	49;0,3 (70.051)
0.02	***	230;0,13 (456.923)	101;0,6 (184.261)	78;0,5 (135.143)	61;0,4 (97.857)	55;0,4 (88.070)
0.025	***	752;0,44 (1504.971)	267;0,18 (540.667)	135;0,10 (266.137)	79;0,6 (143.038)	63;0,5 (107.918)
0.03	***	***	862;0,59 (1788.419)	298;0,23 (597.590)	153;0,13 (305.145)	89;0,8 (170.951)
0.035	***	***	***	957;0,75 (1911.998)	333;0,29 (664.908)	178;0,17 (355.931)
0.04	***	***	***	***	***	***

Table.5.8 *Optimal BRGS plan under G-ZIP distribution for given* $p_1, p_2, \alpha = 0.05, \beta = 0.10, s = 10$ *and* $\omega = 0.09$.

AQL	**LQL (p_2)**					
(p_1)	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
0.005	167;0,3 (180.948)	119;0,2 (126.195)	100;0,2 (106.460)	89;0,2 (94.431)	80;0,2 (84.701)	61;0,1 (62.851)
0.01	409;0,11 (456.721)	241;0,7 (266.445)	160;0,5 (176.768)	120;0,4 (132.239)	93;0,3 (100.703)	83;0,3 (90.078)
0.015	***	740;0,26 (822.420)	358;0,14 (400.650)	218;0,9 (242.516)	139;0,6 (155.153)	110;0,5 (122.328)
0.02	***	***	1442;0,62 (1604.849)	568;0,27 (635.174)	315;0,16 (351.616)	221;0,12 (246.443)
0.025	***	***	***	***	921;0,50 (1020.115)	423;0,25 (473.118)
0.03	***	***	***	***	***	***

*** Sampling plan does not exist

5.1. ILLUSTRATION 1

Suppose that the quality engineer in medical research center wants to run an experiment to make a decision on a product, whether to accept or reject based on BRGS plan under G-ZIP model. If the engineer desired to determine a sampling plan for given sets of strengths AQL and LQL say, $p_1 = 1\%$, $p_2 = 8\%$ with producer's risk (α) 5% and consumers risk (β) 10% and the estimated values of $\omega = 0.05$ and $s = 5$. Under these requirements, from the Table 5.3 one can find value of optimum parameters as $n = 70$, $c_1 = 0$ and $c_2 = 3$ with $ASN = 95.363$.

Execution of the plan:

The BRGS plan can be executed under the G-ZIP conditions is operated as follows:

- **Step 1**: Drawn a random sample of 70 units from the lot and observe the number of non-conforming items (d), in the sample.

- **Step 2**: If there are no nonconforming items is found, then accept the lot. If more than 3 nonconforming items are observed, then reject the lot.
- **Step 3:** If the number of nonconforming items is lies between 0 or 3, then repeat these steps until a decision is made on the same lot.

5.2. ILLUSTRATION 2

Suppose that the quality engineer's interest is to focus on sampling inspection towards submitted lot for a given strength of parameters $p_1 = 0.01, \alpha = 0.05, p_2 = 0.07, \beta = 0.10$ and $\omega = 0.05$. From the Table 5.9, one can find the ASN value of Single Sampling Plan (SSP) under the conditions of G-ZIP distribution is determined for different values of $s = 5$ and 10 respectively. Thus, the optimum of ASN values are $(s = 5, ASN = 318)$ and $(s = 10, ASN = 166)$. But, the Bayesian RGS plan under the G-ZIP distribution for the values of prior information $s = 5$ and 10 respectively, can be determined optimum of ASN values $(s = 5, ASN = 108.99)$ and $(s = 10, ASN = 77.69)$. Therefore, this illustration clearly shows that the BRGS plan is more efficient then the BSSP under the G-ZIP distribution. Further, we made a comparative study on the result of RGS plan with the SSP under the conditions of G-ZIP distribution in terms of ASN values. Obviously, a sampling plan having smaller ASN would be more preferable. Table.5.9 shows the values of ASNs for the RGS plan under G-ZIP distribution with sample size of the SSP under the conditions of G-ZIP distribution for some selected combinations of AQL and LQL values. Here, it is considered only two values of the shape parameter with the different values of $\omega = 0.001, 0.01, 0.05, 0.09$ are given in table.

Table 5.9 *ASN values of the RGS Plan and SSP under the conditions of G-ZIP distribution for specified* p_1, p_2, $\alpha = 0.05$ *and* $\beta = 0.10$.

			s = 5		s = 10	
ω	p_1	p_2	SSP	RGS	SSP	RGS
0.001	0.005	0.05	191	104.85	126	69.55
	0.005	0.06	124	64.21	105	58.38
	0.01	0.06	264	118.11	162	82.12
	0.01	0.07	166	74.61	115	70.68
	0.015	0.07	491	101.05	186	100.54
	0.015	0.08	276	88.42	142	61.72
	0.02	0.08	1432	118.46	204	88.01
	0.02	0.09	452	78.74	145	78.18
	0.025	0.09	***	105.30	236	110.31
	0.025	0.1	***	94.62	277	70.36
0.01	0.005	0.05	198	106.45	108	99.83
	0.005	0.06	129	66.94	93	59.93
	0.01	0.06	308	118.69	166	83.28
	0.01	0.07	203	76.32	118	71.38
	0.015	0.07	567	101.73	191	99.64
	0.015	0.08	311	88.83	146	87.18
	0.02	0.08	***	116.84	231	87.18
	0.02	0.09	535	103.85	167	77.5
	0.025	0.09	***	133.85	260	107.15
	0.025	0.1	***	93.44	185	69.75

Table 5.10 *ASN values of the RGS Plan and SSP under the conditions of G-ZIP distribution for specified* p_1, p_2, $\alpha = 0.05$ *and* $\beta = 0.10$.

			s = 5		s = 10	
ω	p_1	p_2	SSP	RGS	SSP	RGS
0.05	0.005	0.05	242	122.03	154	108.51
	0.005	0.06	202	102.07	129	90.53
	0.01	0.06	579	155.59	226	116.53
	0.01	0.07	318	108.99	166	77.69
	0.015	0.07	***	140.47	273	125.82
	0.015	0.08	716	115.45	193	87.15
	0.02	0.08	***	191.33	355	135.14
	0.02	0.09	***	124.85	233	97.86
	0.025	0.09	***	241.03	459	143.04
	0.025	0.1	***	152.96	285	107.92
0.09	0.005	0.05	842	260.46	332	180.95
	0.005	0.06	515	183.29	234	126.2
	0.01	0.06	***	985.03	475	266.45
	0.01	0.07	***	400.36	337	176.77
	0.015	0.07	***	***	***	400.65
	0.015	0.08	***	1726.47	***	242.52
	0.02	0.08	***	***	***	635.17
	0.02	0.09	***	***	***	351.62
	0.025	0.09	***	***	***	1020.12
	0.025	0.1	***	***	***	473.12

From Table 5.9 and 5.10, it can be clearly observed that the ASN value of the proposed plan is significantly lower than that of the existing Bayesian Single Sampling Plan for all combinations of AQL and LQL values. Furthermore, it is noteworthy that the RGS plan under the G-ZIP distribution is more economical than the Single Sampling Plan in terms of ASN. This suggests that the plan provides the desired protection with minimal inspection, thus greatly reducing the inspection costs.

6. COMPARATIVE ANALYSIS ON SAMPLING PLANS

In garment manufacturing industries, where apparel brands source garments are produced in bulk quantity, sampling inspection is an essential step before placing the bulk order. The sampling process covers garment fit checking, fabric and trims quality checking, approval value-added processes, and approval of complete finished garment. The primary objective of the sampling plan is to safeguard from the risk of making a wrong decision on the part of the manufacturer and the consumers. For instance, the quality control personal wants to run an experiment to make a decision on the quality of the shirts to decide whether the whole lot should be delivered to customers or not based on sampling inspection. Suppose that the garments company desire to determine the sampling plan for a lot consisting of 9000 pieces of shirts with the acceptance quality level is 1% and limiting quality level is 7%. Further, the experimenter is fixed the consumer's and producer's risk level as 5% and 10% with the estimated value of $s = 5$. In order to obtain the comparative study, the values of $p_a(p)$ of the RGS plan under the conditions of GP, ZIP and G-ZIP are given in Table.6.1 for the various values of p.

For instance, the strength of the plan is specified as $p_1 = 0.015, \alpha = 0.05, p_2 = 0.09, \beta = 0.10$ and $s = 10$, it is determined that the RGS plan under the conditions of GP distribution is $(40, 2)$, ZIP distribution are $(33,2), (41,2)$ and for the various fixed parameter $\omega = 0.01,0.05,0.09$ respectively. Corresponding to this strength, the RGS plan under the conditions of G-ZIP distribution are determined for different values of ω. The values of ω and the corresponding RGS Plans are ($\omega = 0.01, n = 37, c = 2$), ($\omega = 0.05, n = 54, c = 3$) and ($\omega = 0.09, n = 139, c = 6$). The value of $P_a(p)$ of these given sampling plans are calculated and are given in Table 4.10. All the RGS plan under the conditions of G-ZIP distribution have been determined for the same strength. It should be noted that the G-ZIP RGS plan enables to take the decision about the current lot considering the past history of the production process. But, the ZIP RGS plan takes decision based on the current sample information only.

From the Table 6.1, it can be observed that all the given sampling plans have higher $P_a(p)$ for lower values of p and have sudden drop in $P_a(p)$ for higher values of p. It indicates that all the given sampling plans, in general, it protects the producer's interest against good quality lots and safeguard the consumer against poor quality lots. In addition, the RGS plan under the conditions of G-ZIP distribution dominate the non-Bayesian ZIP RGS plan and Bayesian GP SSP invariably at all values of p below $p_1 = 0.040$. It shows that the G-ZIP RGS plan provides a better protection to the producer from the risk of rejecting the lots of good quality compared to the non-Bayesian ZIP RGS plan and Bayesian GP RGS plan. For instance, in the case of GP RGS plan, corresponding to $p = 0.01$, the probability of acceptance value is (0.986) and ZIP SSP corresponding to $p = 0.01$, the probability of acceptance values is $0.991, 0.997$ and 0.997 for the various parameter $\omega = 0.01,0.05,0.09$ respectively. But, under the G-ZIP case, for the same values of ω, the probability of acceptance values corresponding to $p = 0.01$ are 0.997, 0.992 and 0.997. It is seen that G-ZIP RGS plan accept lots having lower fraction nonconforming with higher probability.

Table 6.1 *Values of OC functions of GP RGS, ZIP RGS and G-ZIP RGS sampling plans for the given strength* $p_1 = 0.015, \alpha = 0.05, p_2 = 0.09, \beta = 0.10$ *and* $s = 10$.

model		GP RGS	ZIP RGS			G-ZIP RGS		
parameters	ω	-	0.01	0.05	0.09	0.01	0.05	0.09
	n	40	33	41	88	37	54	139
	c	2	2	2	4	2	3	6
Lot fraction nonconforming (p)	0.007	0.995	0.997	0.999	1.000	0.999	0.998	1.000
	0.010	0.986	0.991	0.997	0.997	0.997	0.992	0.997
	0.012	0.976	0.984	0.993	0.992	0.994	0.984	0.989
	0.015	0.954	0.969	0.983	0.973	0.985	0.963	0.952
	0.020	0.896	0.926	0.946	0.879	0.956	0.898	0.781
	0.030	0.713	0.770	0.768	0.508	0.819	0.663	0.376
	0.035	0.606	0.667	0.640	0.362	0.714	0.533	0.271
	0.040	0.504	0.561	0.511	0.268	0.599	0.420	0.209
	0.045	0.412	0.460	0.399	0.208	0.488	0.331	0.171
	0.050	0.334	0.372	0.310	0.171	0.389	0.263	0.147
	0.055	0.269	0.297	0.242	0.146	0.308	0.212	0.132
	0.060	0.217	0.237	0.193	0.130	0.243	0.175	0.121
	0.650	0.176	0.189	0.158	0.118	0.192	0.148	0.113
	0.075	0.117	0.121	0.113	0.105	0.124	0.112	0.103
	0.080	0.096	0.098	0.099	0.100	0.101	0.100	0.100

Table 6.2 *Values of producer, consumer and total sum of risk for the specified strength with* $s = 10$ *of the optimum sampling plan.*

Model	ω	α (%)	β (%)	$\alpha+\beta$ (%)
GP RGS	-	1.41	21.75	23.15
ZIP RGS	0.01	0.91	23.68	24.58
	0.05	0.34	19.34	19.67
	0.09	0.26	12.99	13.25
G-ZIP RGS	0.01	0.3	24.28	24.58
	0.05	0.79	17.53	18.32
	0.09	0.27	12.06	12.32

The probability of acceptance corresponding to $p = 0.06$ for GP RGS plan is 0.217 and ZIP RGS plans are $(0.237, 0.193, 0.130)$ for the parameter $\omega = 0.01, 0.05, 0.09$ respectively. But, under the G-ZIP case, for the same values of ω, the probability of acceptance corresponding to $p = 0.06$ are $(0.243, 0.175, 0.121)$. It shows that G-ZIP RGS plan with moderately higher value of ω accepts the lots having higher fraction nonconforming with lower probability, which also protects consumer against accepting poor quality lots.

Figure.1 are presents the OC curves of G-ZIP RGS plan along with the OC curves of GP and ZIP RGS plans based on Table 6.1. From these Figures, it can be observed that all the three plans of OC curves are in desirable shape with a swell at lower values of p and a sudden drop at higher values of p. However, particularly, the G-ZIP RGS plan has the probability of acceptance is large for good quality lots and is small for bad quality lots, which indicate that these RGS plan ensure protection to both producer and consumer. Further, when the value of ω becomes large, the proposed plan was performed better than the classical RGS plan under the ZIP distribution and the GP RGS plan. From these results, the garments company can be obtained that the proposed BRGS plan will give the optimum sampling plan for the desired quality levels as well as safeguard both the producers and consumers instead of tradition sampling plan.

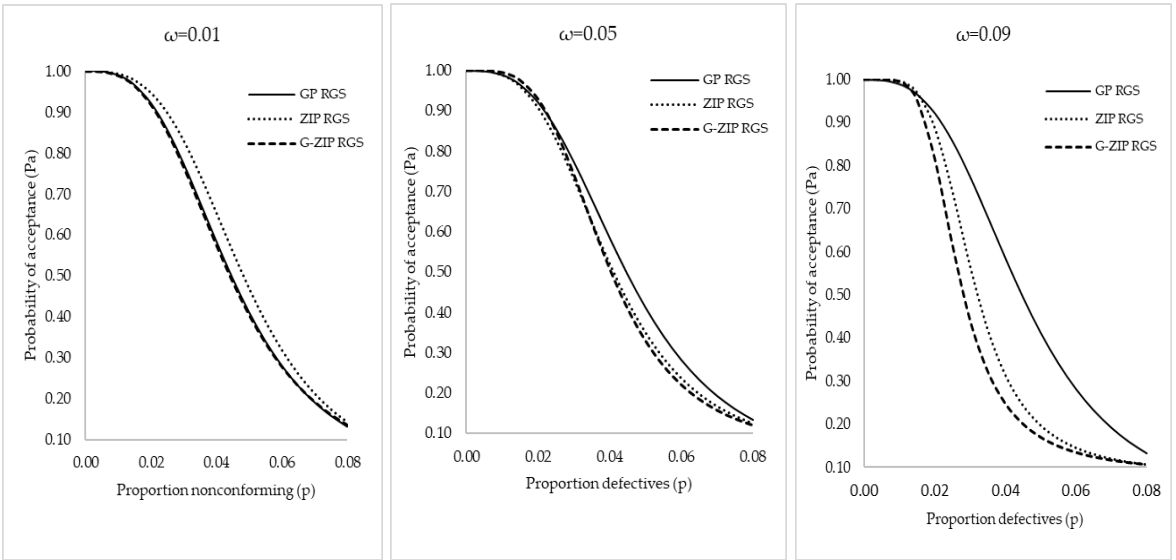

Figure 1: *Comparative analysis of operating characteristic curves*

7. Conclusion

In this article, we have developed a Repetitive Group Sampling plan based on the Gamma-Zero Inflated Poisson (G-ZIP) distribution from a Bayesian perspective. This proposed sampling plan is particularly useful when production processes are well monitored and result in a significant number of defect-free products. Further, the plan incorporates historical production data, allowing for the consideration of inherent or natural variability in the process. The article also provides a comprehensive procedure for designing and selecting plan parameters using the two-point approach on the Operating Characteristic (OC) curve. Additionally, the tables are constructed for some specified strengths $(p_1, p_2, \alpha, \beta)$ for the limited number of values ω and s. From the illustrations, the proposed sampling plan has the better performance measures than over some existing sampling plans. The proposed sampling plan can be applied in various industries Including Electronic device manufacturing industries, medicine research centers and food industries etc. It is strongly suggested that the proposed plan be utilized in the industries for lot sentencing and products determination. The study opens up avenues for further research in the area of Bayesian group sampling. Future investigations could explore the integration of advanced statistical techniques, such as Markov chain Monte Carlo methods, to enhance the precision and efficiency of the proposed methodology. Additionally, the application of the methodology to specific manufacturing industries and the development of decision support tools could be explored to facilitate practical implementation.

References

[1] R. Sherman, (1965). Design and evaluation of a Repetitive Group Sampling Plan,*Technometrics,* 7 (1), 11–12.

[2] V. Soundararajan and M. Ramaswamy, (1986). Procedures and tables for construction and selection of Repetitive Group Sampling Plan, *The QR Journal,* 13 (1), 19-21.

[3] S. Balamurali, H. Park, C.-H. Jun, K.-J. Kim and J. Lee, (2005). Designing of Variables Repetitive Group Sampling Plan involving minimum Average Sample Number, *Communications in Statistics - Simulation and Computation,* 3 (34), 799-809.

[4] G. Kannan, P. Jeyadurga and S. Balamurali, (2022). Economic design of repetitive group sampling plan based on truncated life test under Birnbaum–Saunders distribution, *Communications in Statistics-Simulation and Computation,* 51 (12), 7334-7350.

[5] D. Lambert, (1992). Zero-inflated Poisson regression, with an application to defects in manufacturing, *Technometrics,* 34 (1), 1-14.

[6] D. Bohning, E. Dietz, P. Schlattmann, L. Mendonca and U. Kirchner, (1999). The zero-inflated Poisson model and the decayed, missing and filled teeth index in dental epidemiology, *ournal of the Royal Statistical Society Series A: Statistics in Society,* 162 (2), 195-209.

[7] M. Xie, B. He and T. Goh, (2001). Zero-inflated Poisson model in statistical process control, *Computational statistics & data analysis,* 38 (2), 191-201.

[8] J. Rodrigues, Bayesian analysis of zero-inflated distributions, (2003). *Communications in Statistics-Theory and Methods,* 32 (2), 281-289.

[9] H. Naya, J. I. Urioste, Y. M. Chang, M. Rodrigues-Motta, R. Kremer and D. Gianola, (2008). A comparison between Poisson and zero-inflated Poisson regression models with an application to number of black spots in Corriedale sheep, *Genetics Selection Evolution,* 40, 1-16.

[10] C. H. Sim and M. H. Lim, (2008). Attribute charts for zero-inflated processes, *Communications in Statistics—Simulation and Computation,* 37 (7), 1440-1452.

[11] J. Yang, M. Xie and T. N. Goh, (2011). Outlier identification and robust parameter estimation in a zero-inflated Poisson model, *Journal of Applied Statistics,* 38(2), 421-430.

[12] A. Mussidaa, U. Gonzales-Barron and F. Butler, (2011). Operating characteristic curves for single, double and multiple fraction defective sampling plans developed for Cronobacter in powder infant formula, *Procedia Food Science,* 1, 979-986.

[13] A. Loganathan and K. Shalini, (2014). Determination of Single Sampling Plans by attributes under the conditions of Zero-Inflated Poisson distribution, *Communications in Statistics - Simulation and Computation,* 43 (3), 538-548.

[14] G. Rao and M. Aslam, (2017). Resubmitted lots with single sampling plans by attributes under the conditions of zero-inflated Poisson distribution, *Communications in Statistics-Simulation and Computation,* 46 (3), 1814-1824.

[15] A. Hald, (1981). Statistical Theory of Sampling Inspection by Attributes, New York: Academic Press.

[16] T. Calvin, (1990). How and When to Perform Bayesian Acceptance Sampling, *American Society for Quality Control, Wisconsin,* 7.

[17] K. E. Case and J. B. Keats, (1982). On the selection of a prior distribution in Bayesian Acceptance Sampling, *Journal of Quality Technology,* 14 (1), 10-18.

[18] D. Guthrie and M. Johns, (1959). Bayes acceptance sampling procedures for large lots, *The Annals of Mathematical Statistics,* 30 (4), 896-925.

[19] R. Vijayaraghavan, K. Rajagopal and A. Loganathan, (2008). A procedure for selection of a gamma-Poisson single sampling plan by attributes, *Journal of Applied Statistics,* 35 (2), 149-160.

[20] K. Rajagopal, A. Loganathan and R. Vijayaraghavan, (2009). Selection of Bayesian Single Sampling Attributes Plans Based on Polya Distribution, *Stochastics and Quality Control,* 24 (2), 179-193.

[21] E. G. Schilling, (1982). Revised Attributes Acceptance Sampling Standard—ANSI/ASQC Z1. 4, *Journal of Quality Technology,* 14 (4), 215-219.

[22] W. Guenther, (1969). Use of the binomial, hypergeometric and Poisson tables to obtain sampling plans, *Journal of Quality Technology,* 1, 105–109.