PARTIAL ASYMPTOTIC ANALYSIS METHOD FOR TWO-CLASS RETRIAL QUEUE WITH CONSTANT RETRIAL RATE*

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Abstract

In the paper, a single-server retrial queueing system with two types of arrivals and a constant retrial policy is considered as a mathematical model of a multimodal telecommunication network. Service, inter-arrival and inter-retrial times have exponential distributions. The constant retrial policy means that only the first customer from an orbit performs an attempt to get a service. The method of partial asymptotic analysis under a condition of a heavy load of one class of customers is proposed. The formula for the asymptotic characteristic function of the stationary marginal probability distribution of the number of customers of one class is derived. In addition, the system stability conditions are discussed. Some numerical examples are presented.

Keywords: two-class retrial queueing system, constant retrial policy, partial asymptotic analysis, heavy load

1. Introduction

Modern telecommunication networks have several types of transmitted information (text, sound, image, service information, etc.). The transmission and processing of heterogeneous data need more complex preliminary analysis. An example of heterogeneous networks is the multimodal system [1, 2]. Most studies of multimodal systems are based on simulation. Although various mathematical models of such systems have been proposed, their analytical study is almost not carried out because of the need to study multidimensional random processes. For networks optimizing and planing, assessing their reliability and efficiency, queuing theory is usually used [3].

Retrial queueing systems (RQ or queueing system with repeated calls) [4, 5] are new class of queuing models widely applied in various communication systems [6, 7]. In spite of the large number of studies of retrial queueing systems in various configurations, heterogeneous models are weak investigated. Retrial queues with several types of customers (and several orbits too) are called as multiclass retrial queues and considered in [8, 9, 10, 11, 12, 13, 14]. Most of the cited papers are devoted only the stability analysis, while probability distributions or even means of processes under study are hardly investigated.

In the paper, we propose an partial asymptotic analysis method for two-dimensional Markov process analyzing. The asymptotic analysis method [15, 16] is applied for study of various classes

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of queueing systems. It allows us to solve Kolmogorov's equations under some asymptotic (i.e. limit) condition when they cannot be solved directly. But usually this method is used for one-dimensional processes under study. Here we try to apply the asymptotic analysis approach for multiclass queueing models. Also we demonstrate the method applying for retrial queues with constant retrial policy for the first time.

The rest of the paper is organized as follows. In Section 2, the mathematical model and the process under study are described. In Section 2.1, Kolmogorov equations are written and expressions for the stationary probabilities of the server states are obtained. In Section 2.2, there are discussions about stability and partial stability areas of the model. Section 3 is devoted to applying the original partial asymptotic analysis method under a heavy load of one class of customers. In Section 4, some numerical results are presented. Section 5 consists of some conclusions.

2. Mathematical Model

We consider a retrial queueing system with two classes of customers. The n-th class customer comes to the system according to Poisson arrival process with parameter λ_n, where $n = 1, 2$. There is one server. If the server is idle, an arrival (the n-th class customer) starts its servicing during the exponentially distributed random time with rate μ_n. If the arrival finds the server busy, it goes to the corresponding orbit and waits for a random time exponentially distributed with rate σ_n there. There are two orbits for each class customers. We consider the constant retrial policy, which means that only the first customer from an orbit has access to the server. The model structure is presented in Figure 1.

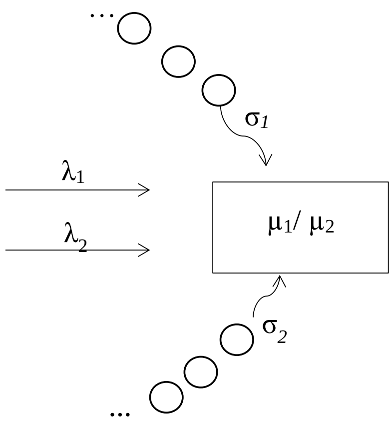

Figure 1: *Two-class retrial queueing system with constant retrial rate*

Let us denote the random processes of the number of customers in the n-th orbit by $i_n(t)$, where $n = 1, 2$. Process $k(t)$ determines the state of the server in the following way: $k(t) = 0$ if the server is free and $k(t) = n$ if the n-th class customer is servicing. Process $\{k(t), i_1(t), i_2(t)\}$ is a three-dimensional continuous-time Markov chain.

2.1. Kolmogorov equations

Denote the probability that the server has state k and there are i_1 customers in the first orbit and i_2 customers in the second orbit at time t by $P\{k(t) = k, i_1(t) = i_1, i_2(t) = i_2\} = P(k, i_1, i_2, t)$. Let us write the following system of Kolmogorov equations for $P(k, i_1, i_2, t)$:

$$
\begin{cases}
\frac{\partial P(0,i_1,i_2,t)}{\partial t} = -(\lambda_1+\lambda_2+\delta_{i_1}\sigma_1+\delta_{i_2}\sigma_2)P(0,i_1,i_2,t)+ \\
+\mu_1 P(1,i_1,i_2,t)+\mu_2 P(2,i_1,i_2,t), \\
\frac{\partial P(1,i_1,i_2,t)}{\partial t} = -(\lambda_1+\lambda_2+\mu_1)P(1,i_1,i_2,t)+\lambda_1 P(0,i_1,i_2,t)+ \\
+\sigma_1 P(0,i_1+1,i_2,t)+\lambda_2 P(1,i_1,i_2-1,t)+\lambda_1 P(1,i_1-1,i_2,t), \\
\frac{\partial P(2,i_1,i_2,t)}{\partial t} = -(\lambda_1+\lambda_2+\mu_2)P(2,i_1,i_2,t)+\lambda_2 P(0,i_1,i_2,t)+ \\
+\sigma_2 P(0,i_1,i_2+1,t)+\lambda_1 P(2,i_1-1,i_2,t)+\lambda_2 P(2,i_1,i_2-1,t),
\end{cases}
\quad (1)
$$

where $\delta_i = \{0 \text{ if } i=0; 1 \text{ if } i \neq 0\}$ is Kronecker delta.

Then we obtain the following balance equations in steady state

$$
\begin{cases}
-(\lambda_1+\lambda_2+\delta(i_1)\sigma_1+\delta(i_2)\sigma_2)P(0,i_1,i_2)+ \\
+\mu_1 P(1,i_1,i_2)+\mu_2 P(2,i_1,i_2)=0, \\
-(\lambda_1+\lambda_2+\mu_1)P(1,i_1,i_2)+\lambda_1 P(0,i_1,i_2)+ \\
+\sigma_1 P(0,i_1+1,i_2)+\lambda_2 P(1,i_1,i_2-1)+\lambda_1 P(1,i_1-1,i_2)=0, \\
-(\lambda_1+\lambda_2+\mu_2)P(2,i_1,i_2)+\lambda_2 P(0,i_1,i_2)+ \\
+\sigma_2 P(0,i_1,i_2+1)+\lambda_1 P(2,i_1-1,i_2)+\lambda_2 P(2,i_1,i_2-1)=0.
\end{cases}
\quad (2)
$$

Let us introduce the partial characteristic functions as follows

$$
H(k,u_1,u_2) = \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty} e^{ju_1 i_1}\cdot e^{ju_2 i_2} P(k,i_1,i_2),
$$

$$
h_2(u_1) = \sum_{i_1=0}^{\infty} e^{ju_1 i_1} P(0,i_1,0), \quad h_1(u_2) = \sum_{i_2=0}^{\infty} e^{ju_2 i_2} P(0,0,i_2).
$$

Then Equations (2) are rewritten as

$$
\begin{cases}
-(\lambda_1+\lambda_2+\sigma_1+\sigma_2)H(0,u_1,u_2)+ \\
+\mu_1 H(1,u_1,u_2)+\mu_2 H(2,u_1,u_2)+\sigma_1 h_1(u_2)+\sigma_2 h_2(u_1)=0, \\
(\lambda_1(e^{ju_1}-1)+\lambda_2(e^{ju_2}-1)-\mu_1)H(1,u_1,u_2)+ \\
+(\lambda_1+\sigma_1 e^{-ju_1})H(0,u_1,u_2)-\sigma_1 e^{-ju_1}h_1(u_2)=0, \\
(\lambda_1(e^{ju_1}-1)+\lambda_2(e^{ju_2}-1)-\mu_2)H(2,u_1,u_2)+ \\
+(\lambda_2+\sigma_2 e^{-ju_2})H(0,u_1,u_2)-\sigma_2 e^{-ju_2}h_2(u_1)=0.
\end{cases}
\quad (3)
$$

By summing up all equations of System (3), we obtain an additional equation

$$
\begin{gathered}
(\sigma_1(e^{-ju_1}-1)+\sigma_2(e^{-ju_2}-1))H(0,u_1,u_2)+ \\
+(\lambda_1(e^{ju_1}-1)+\lambda_2(e^{ju_2}-1))(H(1,u_1,u_2)+H(2,u_1,u_2))+ \\
+\sigma_1(1-e^{-ju_1})h_1(u_2)+\sigma_2(1-e^{-ju_2})h_2(u_1)=0.
\end{gathered}
\quad (4)
$$

Let us denote the stationary probability of the server states by $R_k = H(k,0,0)$ for $k=0,1,2$ and the stationary probability that the server is idle and the corresponding orbit is empty by $r_1 = h_1(0)$ and $r_2 = h_2(0)$.

From Equations (3)–(4), the following equations for probabilities R_k, r_n can be obtained

$$
\begin{cases}
-(\lambda_1+\lambda_2+\sigma_1+\sigma_2)R_0+\mu_1 R_1+\mu_2 R_2 \sigma_1 r_1+\sigma_2 r_2=0, \\
-\mu_1 R_1+(\lambda_1+\sigma_1)R_0-\sigma_1 r_1=0, \\
-\mu_2 R_2+(\lambda_2+\sigma_2)R_0-\sigma_2 r_2=0, \\
-\sigma_2 R_0+\lambda_2(R_1+R_2)\sigma_2 r_2=0, \\
-\sigma_1 R_0+\lambda_1(R_1+R_2)\sigma_1 r_1=0.
\end{cases}
\quad (5)
$$

Taking into account the normalization condition $R_0 + R_1 + R_2 = 1$ in Equations (5), it is easy to derive expressions for probabilities R_k, r_n.

$$R_1 = \frac{\lambda_1}{\mu_1}, \quad R_2 = \frac{\lambda_2}{\mu_2}, \quad R_0 = 1 - \frac{\lambda_1}{\mu_1} - \frac{\lambda_2}{\mu_2},$$
$$r_1 = \frac{1}{\sigma_1}\left[\sigma_1 - (\lambda_1 + \sigma_1)\left(\frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2}\right)\right], \tag{6}$$
$$r_2 = \frac{1}{\sigma_2}\left[\sigma_2 - (\lambda_2 + \sigma_2)\left(\frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2}\right)\right].$$

2.2. Stability

From Expressions (6), due to non-negative values of probabilities, it can be obtained the necessary conditions of the system stability:

$$\frac{\lambda_1}{\mu_1} - \frac{\lambda_2}{\mu_2} < 1,$$
$$\sigma_1 > \lambda_1 \frac{R_1 + R_2}{R_0}, \quad \sigma_2 > \lambda_2 \frac{R_1 + R_2}{R_0} \tag{7}$$

that matches with the results [9], where it is proved that inequalities (7) are necessary and sufficient conditions of system stability.

Also, we can rewrite inequalities (7) as

$$R_1 + R_2 < \frac{\sigma_1}{\sigma_1 + \lambda_1}, \quad R_1 + R_2 < \frac{\sigma_2}{\sigma_2 + \lambda_2}. \tag{8}$$

where $R_1 = \lambda_1/\mu_1$ and $R_2 = \lambda_2/\mu_2$ have the meaning of the corresponding class load parameters.

In other words, failure to satisfy condition (8) entails negative values of stationary probabilities r_n, so there is no steady state for the customers of the corresponding class (i.e., the number of customers in the orbit tends to infinity).

Note that the existence of steady-state distributions in the considered queueing system with constant retrial policy depends on values of all model parameters, while a classical retrial queue stability does not depend on values of retrial rates.

In addition, we demonstrate the stability and instability areas of the model through numerical examples. First, we denote the maximum possible (unachievable) values of each class arrival and retrial parameters when the system is in steady state by S_n and G_n, respectively. Sometimes, parameters S_1 and S_2 are called throughput. In other words, parameters S_n and G_n are a constraint on system rates λ_n and σ_n, if they are not met, inequalities (8) are not satisfied and the system is not in steady state. From expressions (6), we can obtain that

$$G_1 = \frac{\lambda_1(\lambda_1\mu_2 + \lambda_2\mu_1)}{\mu_1\mu_2 - \lambda_1\mu_2 - \lambda_2\mu_1}, \quad G_2 = \frac{\lambda_2(\lambda_1\mu_2 + \lambda_2\mu_1)}{\mu_1\mu_2 - \lambda_1\mu_2 - \lambda_2\mu_1} \tag{9}$$

and

$$S_1 = \left(\frac{\lambda_2}{\mu_2} + \frac{\sigma_1}{\mu_1}\right)\frac{\mu_1}{2}\left\{\sqrt{1 + 4\frac{\frac{\sigma_1}{\mu_1}\left(1 - \frac{\lambda_2}{\mu_2}\right)}{\left(\frac{\lambda_2}{\mu_2} + \frac{\sigma_1}{\mu_1}\right)^2}} - 1\right\},$$
$$S_2 = \left(\frac{\lambda_1}{\mu_1} + \frac{\sigma_2}{\mu_2}\right)\frac{\mu_2}{2}\left\{\sqrt{1 + 4\frac{\frac{\sigma_2}{\mu_2}\left(1 - \frac{\lambda_1}{\mu_1}\right)}{\left(\frac{\lambda_1}{\mu_1} + \frac{\sigma_2}{\mu_2}\right)^2}} - 1\right\}. \tag{10}$$

So, the system stability conditions can be written as $\sigma_n > G_n$ or $\lambda_n < S_n$.

Then we demonstrate the areas of the system stability and instability in numerical examples. Let the system parameters be $\mu_1 = 1$, $\mu_2 = 2$. Expressions (9) are illustrated in Figure 2 depending on values of σ_n for arrival rates $\lambda_1 = 0.1$, $\lambda_2 = 0.3$. Expressions (10) are shown in Figure 3 depending on values of λ_n for $\sigma_1 = 0.1$, $\sigma_2 = 0.2$.

As we can see in Figures 3,2, there are four zones:

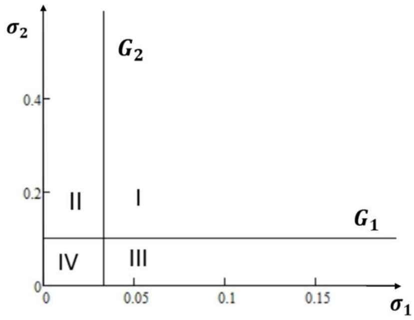

Figure 2: *Stability areas vs.* σ_1, σ_2

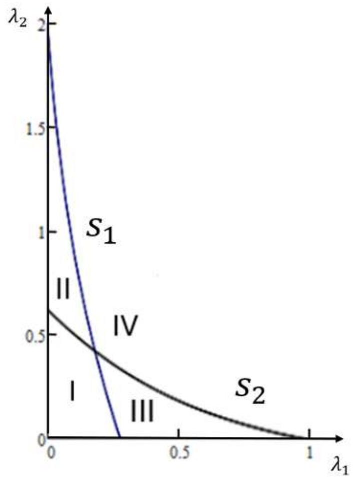

Figure 3: *Stability areas vs.* λ_1, λ_2

- zone I – stability of both orbits (both $\lambda_n < S_n$ and $\sigma_n > G_n$);
- zone II – stability of the first orbit, while the number of customers on the second orbit tends to infinity ($\lambda_1 < S_1$ and $\sigma_1 > G_1$, but $\lambda_2 > S_2$ and $\sigma_2 < G_2$);
- zone III – stability of the second orbit, while the number of customers on the first orbit tends to infinity ($\lambda_2 < S_2$ and $\sigma_2 > G_2$, but $\lambda_1 > S_1$ and $\sigma_1 < G_1$);
- zone IV – instability of both orbits (both $\lambda_n > S_n$ and $\sigma_n < G_n$).

3. Method

Equations (3)–(4) contain five unknown functions of two variables, so the direct solution is impossible. We propose applying an asymptotic analysis approach, namely, the method of partial asymptotic analysis under the condition of a heavy load of one class of customers to Equations (3)–(4) solving. Let us choose a condition of a heavy load of the second class of customers, so $\lambda_2 \to S_2$. Graphically, we can demonstrate the asymptotic condition as the red curve in Figure 4. Further we will derive the steady state asymptotic probability distribution of the number of customers in the first orbit. Obviously, the asymptotic results must be closer to the exact probability distribution with λ_2 closer to S_2.

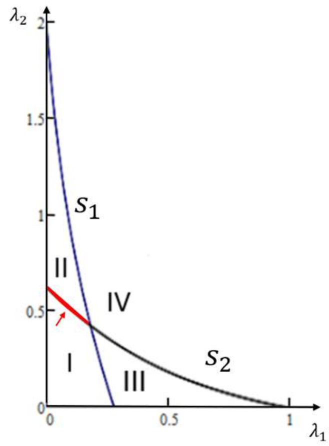

Figure 4: *Asymptotic condition (red curve)*

3.1. Asymptotic equations

For System (3)-(4) solving, we introduce an infinitesimal parameter ε. Then the asymptotic condition can be rewritten as $\lambda_2 = S_2(1-\varepsilon)$, where $\varepsilon \to 0$. Also we introduce the following notations

$$\begin{aligned} u_2 = \varepsilon w, \quad H(k, u_1, u_2) = F(k, u_1, w, \varepsilon), \\ h_1(u_2) = f_1(w, \varepsilon), \quad h_2(u_1) = \varepsilon f_2(u_1). \end{aligned} \tag{11}$$

Substituting (11) into Equations (3)–(4), we obtain

$$\begin{cases} -(\lambda_1 + S_2(1-\varepsilon) + \sigma_1 + \sigma_2)F(0, u_1, w, \varepsilon) + \\ +\mu_1 F(1, u_1, w, \varepsilon) + \mu_2 F(2, u_1, w, \varepsilon) + \sigma_1 f_1(w, \varepsilon) + \sigma_2 \varepsilon f_2(u_1) = O(\varepsilon^2), \\ (\lambda_1(e^{ju_1} - 1) + S_2(1-\varepsilon)(e^{jw\varepsilon} - 1) - \mu_1)F(1, u_1, w, \varepsilon) + \\ +(\lambda_1 + \sigma_1 e^{-ju_1})F(0, u_1, w, \varepsilon) - \sigma_1 e^{-ju_1} f_1(w, \varepsilon) = O(\varepsilon^2), \\ (\lambda_1(e^{ju_1} - 1) + S_2(1-\varepsilon)(e^{jw\varepsilon} - 1) - \mu_2)F(2, u_1, w, \varepsilon) + \\ +(S_2(1-\varepsilon) + \sigma_2 e^{-jw\varepsilon})F(0, u_1, w, \varepsilon) - \sigma_2 e^{-jw\varepsilon} \varepsilon f_2(u_1) = O(\varepsilon^2), \\ (\sigma_1(e^{-ju_1} - 1) + \sigma_2(e^{-jw\varepsilon} - 1))F(0, u_1, w, \varepsilon) + \\ +(\lambda_1(e^{ju_1} - 1) + S_2(1-\varepsilon)(e^{jw\varepsilon} - 1))(F(1, u_1, w, \varepsilon) + F(2, u_1, w, \varepsilon)) + \\ +\sigma_1(1 - e^{-ju_1}) f_1(w, \varepsilon) + \sigma_2(1 - e^{-jw\varepsilon}) \varepsilon f_2(u_1) = O(\varepsilon^2). \end{cases} \tag{12}$$

Taking into account $\lim\limits_{\varepsilon \to 0} f_1(w, \varepsilon) = r_1$, Equations (12) are written under $\varepsilon \to 0$ as

$$\begin{cases} -(\lambda_1 + S_2 + \sigma_1 + \sigma_2)F(0, u_1, w) + \mu_1 F(1, u_1, w) + \mu_2 F(2, u_1, w) + \sigma_1 r_1 = 0, \\ (\lambda_1(e^{ju_1} - 1) - \mu_1)F(1, u_1, w) + (\lambda_1 + \sigma_1 e^{-ju_1})F(0, u_1, w) - \sigma_1 e^{-ju_1} r_1 = 0, \\ (\lambda_1(e^{ju_1} - 1) - \mu_2)F(2, u_1, w) + (S_2 + \sigma_2)F(0, u_1, w) = 0, \\ \sigma_1(e^{-ju_1} - 1)F(0, u_1, w) + \lambda_1(e^{ju_1} - 1)(F(1, u_1, w) + F(2, u_1, w)) + \\ +\sigma_1(1 - e^{-ju_1}) r_1 = 0. \end{cases} \tag{13}$$

To obtain the marginal probability distribution, we will set $w = 0$ in System (13) and denote $H^{(1)}(k, u) = F(k, u_1, 0)$.

$$\begin{cases} -(\lambda_1 + S_2 + \sigma_1 + \sigma_2)H^{(1)}(0, u) + \mu_1 H^{(1)}(1, u) + \mu_2 H^{(1)}(2, u) + \sigma_1 r_1 = 0, \\ (\lambda_1(e^{ju} - 1) - \mu_1)H^{(1)}(1, u) + (\lambda_1 + \sigma_1 e^{-ju})H^{(1)}(0, u) - \sigma_1 e^{-ju} r_1 = 0, \\ (\lambda_1(e^{ju} - 1) - \mu_2)H^{(1)}(2, u) + (S_2 + \sigma_2)H^{(1)}(0, u) = 0, \\ \sigma_1 H^{(1)}(0, u) + \lambda_1 e^{ju}(H^{(1)}(1, u) + H^{(1)}(2, u)) + \sigma_1 r_1 = 0. \end{cases} \tag{14}$$

The solution of System (14) is following

$$H^{(1)}(0,u) = \frac{\sigma_1 r_1(\lambda_1(1-e^{ju})+\mu_2)(\mu_1-\lambda_1 e^{ju})}{(\sigma_1\mu_1 - \lambda_1 e^{ju}(\sigma_1+\lambda_1)(\mu_2+\lambda_1(1-e^{ju})) - \lambda_1 e^{ju}(S_2+\sigma_2)(\mu_1+\lambda_1(1-e^{ju}))},$$
$$H^{(1)}(1,u) = \frac{(\lambda_1+\sigma_1 e^{-ju})H^{(1)}(0,u) - \sigma_1 r_1 e^{-ju}}{\lambda_1(1-e^{ju})+\mu_1}, \quad H^{(1)}(2,u) = \frac{(S_2+\sigma_2)}{\lambda_1(1-e^{ju})+\mu_2}H^{(1)}(0,u). \tag{15}$$

In this way, we can obtain the partial characteristic function of the number of customers in the first orbit under a heavy load of the second class customers

$$H^{(1)}(u) = H^{(1)}(0,u) + H^{(1)}(1,u) + H^{(1)}(2,u),$$

then the probability distribution of the number of customers in the first orbit can be approximate as

$$P_1(i_1) \approx \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-jui_1}H^{(1)}(u)du. \tag{16}$$

Note that we can derive the marginal asymptotic probability distribution of the number of customers in the second orbit in the same way.

4. Results

Using Expressions (15), we can be obtained main characteristics of the model under study such as marginal probability distribution of each class customers (16), probabilities of server states (6) or means and high order moments as

$$\mathrm{E}[(i_1(t))^n] = j^{-n}\frac{d^n H^{(1)}(u)}{du}\bigg|_{u=0}.$$

For a demonstration of the results of the asymptotic method, we consider a numerical example. Consider the system parameters have the following values:

$$\lambda_1 = 0.1, \quad \sigma_1 = 5, \quad \sigma_2 = 10, \quad \mu_1 = 1, \quad \mu_2 = 2$$

and the second class of customers makes a heavy load to the system. We introduce a load parameter ρ such as $\lambda_2 = \rho S_2$; under asymptotic condition $\rho \to 1$.

First, we analyze S_1 and S_2 dependence on the parameters of the second class (λ_2 and σ_2) (Figure 5).

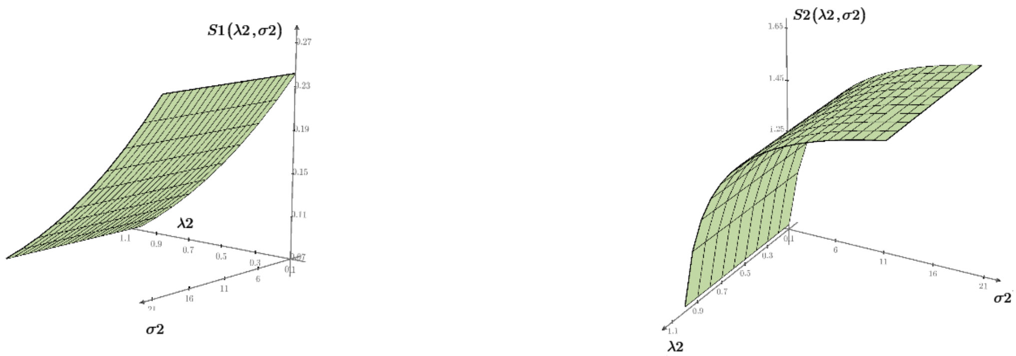

Figure 5: *Values of S_1 and S_2 vs. λ_2 and σ_2*

We can conclude that the first class throughput parameter S_1 decreases with arrival rate λ_2 increases and does not depend on retrial rate σ_2. In addition, the second class throughput parameter S_2 increases with retrial rate increasing.

In Figure 6, the probability distribution of the number of customers of the first class is presented for $\rho = 0.9$. In Table 1, the values of the mean of the number of customers in the first orbit are given for different values of ρ.

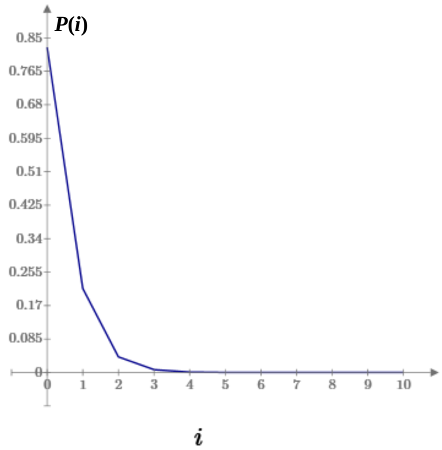

Figure 6: *Probability distribution of the number of customers in the first orbit for $\lambda_2 = 0.9S_2$*

Table 1: *Mean of the number of customers in the first orbit*

ρ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$E\{i_1(t)\}$	1.509	1.365	1.221	1.077	0.933	0.789	0.645	0.501	0.357	0.213

In Figure 7, the stationary probabilities of the server states depending on ρ are presented. As we can see, probability R_1 does not depend on parameter ρ (as on $\lambda_2 = \rho S_2$), it is also proved by formulas (6). Probabilities R_0 and R_2 have linear dependence on ρ.

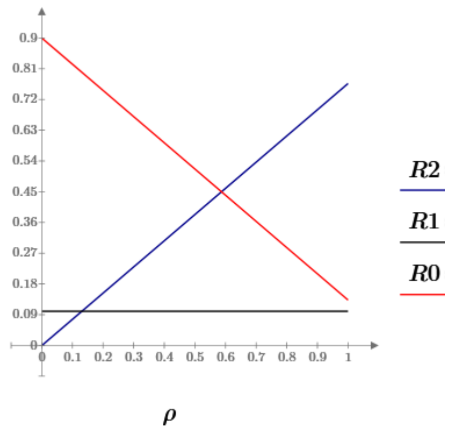

Figure 7: *Probabilities of the server states R_k vs. ρ*

In Figure 8, the stationary probabilities of empty orbits r_n are presented. We can conclude that the probabilities that corresponding orbit and the server are empty decrease ($r_2 \to 0$) with $\rho \to 1$. That can be explained by the fact that the stability boundary is approaching, so the probability of an empty orbit becomes smaller and smaller.

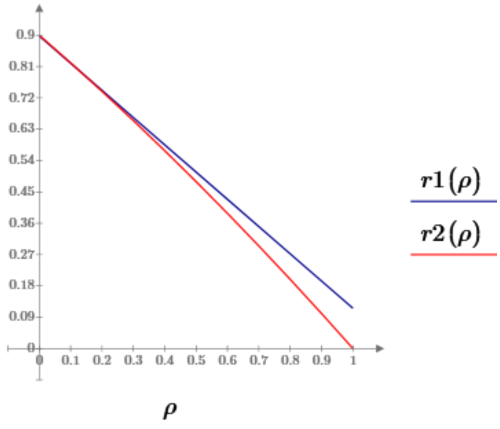

Figure 8: *Probabilities of empty orbits r_n vs. ρ*

5. Conclusions

In this way, we have proposed the partial asymptotic analysis method for the two-class retrial queuing systems with constant retrial rate. During the research, we have derived the asymptotic characteristic function of the number of customers in one orbit under a heavy load of the system by other class customers, from which all marginal characteristics of the model can be found. Obviously, asymptotic formulas will give us more precise results if an arrival process rate tends to its throughput: $\lambda_n \to S_n$. Numerical examples are also presented.

The asymptotic method allows us to derive analytical formulas in case of impossibility of explicit formulas deriving. In this way, in future studies, the proposed methods will be applied for more complex models such as multiclass retrial queue, systems with MMPP arrivals, priority customers, etc. Also, we plan to develop the approach for other asymptotic conditions, for example, an overload of one orbit.

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