

QUEUEING-INVENTORY K-OUT-OF-N SYSTEM WITH HEAVY TAILS

ARYA P S^{1,a}, MANIKANDAN RANGASWAMY^{1,b}, ALEXANDER RUMYANTSEV^{2,3,c}

•
¹ Department of Mathematics, Central University of Kerala,
Periye, Kasaragod, 671320, Kerala, India

² Institute of Applied Mathematical Research, Karelian Research Center of RAS,
Petrozavodsk, Russia

³Petrozavodsk State University, Petrozavodsk, Russia

arya.p.s@cukerala.ac.in^a, mani@cukerala.ac.in^b, ar0@krc.karelia.ru^c

Abstract

In this paper, we study the so-called k -out-of- n queueing-inventory system with a single repair unit, identical elements that are subject to failure, stock of spare elements, and state-dependent replenishment policy. The finite state space Markov chain model is described, and key stationary performance measures are defined. The key focus of this research is on the non-Markov case, in which the random repair and replenishment times may have infinite means, which may affect the positive recurrence of the states of the model. This case is investigated numerically.

Keywords: k -out-of- n : G system, degradation, queueing-inventory model, heavy tails, positive recurrence, generalized semi-Markov processes

1. INTRODUCTION

The reliability analysis of k -out-of- n systems is a topic of considerable interest due to its wide application in various real-world systems, including telecommunications, manufacturing, and power generation. These systems ensure that a system remains operational as long as at least k components out of n are functioning. The complexity of k -out-of- n systems arises from the intricate nature of component failures and the repair mechanisms employed to maintain system functionality.

Over time, various models have been developed to analyze and optimize the reliability of such systems, taking into account component failure rates, repair policies, and replenishment strategies. These aspects were considered in a wide range of applications. Failure analysis and statistical modeling were used in enhancing the system resilience of a large-scale hybrid supercomputer with repair and replenishment of components [1]. Reliability evaluation of power systems using multi-state warm standby components and performance-sharing mechanisms was done in [2]. The repair and replenishment are also applied in industrial applications like solar power management at Cochin International Airport [3], where prompt repair or replacement of malfunctioning equipment is critical in maintaining efficiency, as detailed in [4].

In systems with repair, replenishment orders are crucial for maintaining operational availability, especially in COLD, WARM, and HOT systems. A system is classified as COLD when operational components stop deteriorating after failure until the system is restored, as WARM

when operational components continue to deteriorate at a reduced rate after failure, and as HOT when operational components deteriorate at the same rate regardless of system failure. In the preliminary study conducted by [5], the system performance is optimized through the hiring of a repair server upon the failure of N components. This study is further extended in [6], where they investigate a k -out-of- $n : G$ system with an N -policy replenishment, where components are replaced upon failure, leading to optimized long-run state probabilities, and in [7], which explores k -out-of- n systems with the retrial of failed units, focusing on reliability across cold, warm, and hot standby systems. Incorporating an unreliable server and phase-type repair times, [8] extends this analysis under the (N, T) policy. Recently, [9] analyzes a k -out-of- n repairable system with a server offering single or bulk service to failed components. Through a Continuous-time Markov Chain (CTMC) model, the study evaluates system reliability, server performance, and cost functions, demonstrating the advantages of the single/bulk service model over the traditional single-unit service policy, whereas [10] introduces an innovative repair policy for circular consecutive- k -out-of- $n : F$ systems, prioritizing emergency repairs to prevent system failure while managing ordinary repairs in a queue, and provides valuable insights into minimizing repair costs while enhancing system performance.

Additionally, [11] examines a k -out-of- n system with repair, extending service to external customers through a retrial queue, providing insights into system size probabilities and performance metrics. Emphasizing the significance of general repair time distributions on system reliability, [12] provides a comprehensive probabilistic analysis of a k -out-of- n system, where a mathematical model, along with an algorithm to calculate the reliability function and validate their findings through numerical investigations, which pave the way for future sensitivity analyses of reliability characteristics in such systems.

Traditional models, such as Markov processes and fault tree analysis, have been extensively used to assess the reliability of the k -out-of- n systems. However, the models often rely on the assumption of exponential distributions and the memoryless property, which may only sometimes reflect the true complexity of real-world systems. As highlighted in [13], these assumptions can lead to inaccurate reliability estimates if they involve more complex service time distributions or non-exponential failure and repair rates; there is a need for more sophisticated models in real-world systems. To address this limitation, Generalized Semi-Markov Processes (GSMPs) have been introduced. GSMPs extend classical Markov models by allowing arbitrary, non-exponential distributions for the timing of events. These processes provide a flexible framework where states transition based on event clocks, accommodating the modeling of systems with complex, variable timing behaviors [14, 15, 16]. Simulation-based methods offer a robust tool for analyzing the dynamic behavior of k -out-of- n systems, where complex interactions between failure, repair, and replenishment processes are modeled. For example, [17] utilize matrix-based reliability methods for k -out-of- n systems, revealing challenges in analyzing system reliability due to the statistical dependence between component failures. By extending such approaches, it is possible to capture various performance metrics, such as system downtime, repair time, and cost optimization.

Using simulation, [18] explores the asymptotic insensitivity of k -out-of- n systems, while [19] analyzes their steady-state reliability under full repair. Studies such as [20] examine repairable k -out-of- n systems where some components are suspended during repair, and [21] further considers systems with repairmen taking vacations and a shut-off rule for non-failed components. [22] investigates the optimization of inspection and maintenance schedules for k -out-of- $n : G$ warm standby systems, emphasizing the need to activate standby components effectively.

As systems become more intricate, they require advanced models accounting for non-exponential distributions, time-dependent transitions, and interdependency between components. These demands have led to the increasing use of simulation-based approaches, particularly Discrete-Event Simulation (DES), which is highly flexible and capable of accurately modeling complex systems. The DES approach has been effectively applied in fields such as operations research, healthcare, and manufacturing to evaluate processes involving events like failures, repairs, and maintenance activities over time [23, 24]. Additionally, research has explored the implications of external customer services on system reliability. Studies such as [25] and [26]

examine systems where servers handle component repairs and external customer services. These models introduce new layers of complexity, with performance measures such as server idle time utilization and queue management playing a crucial role in overall system reliability.

Building upon these foundational studies, our work focuses on a k -out-of- $n : G$, WARM system with repair and replenishment orders. We develop a simulation model within the GSMP framework to analyze the system with general distributions for repair times, replenishment orders, and system reliability. The key question in this direction is the positive recurrence of the system states, which, in general, is not guaranteed in advance. We address this question by a numerical study of the simulation model under the heavy-tailed distribution assumptions, which is the main contribution of the present study.

The rest of the paper is arranged as follows: Section 2 provides a detailed description and key performance metrics of the model in the Markovian case, while Section 3 studies the recurrence of the system states (in general case) by constructing a simulation model. The paper ends with a brief conclusion.

2. MODEL DESCRIPTION

The k -out-of- $n : G$ queueing-inventory system has $n > 0$ identical elements, a single repair server, and a finite inventory for storing *spare* elements. The lifetimes of the *working* elements are independent and identically distributed (iid). Depending on the number of (remaining) working elements, the system can be in *failure state* (if less than $k > 0$ elements are working) or in *active state* (otherwise). When the system is in an active state, the lifetimes of the elements follow an exponential distribution with rate $\lambda > 0$, and in the failure state they follow an exponential distribution with rate $\theta > 0$. The defective elements form a queue and the repair process starts when the number of working elements drops to $L < n$. The system assigns iid repair times following an exponential distribution with rate γ and restores repaired elements to a "good as new" condition. The repair stops when the number of working elements increases to H , $L < H < n$. Apart from repair, restoration of the elements in the system also happens due to replenishment. That is, if the number of working elements reduces to N , $k < N < L$, an order for replenishment is placed. The size of the replenishment order depends on the state of the system in which replenishment occurs:

1. In the active system state, we place a fixed order size of $(n - k + 1)$ elements.
2. In the failure system state, the total number of working elements is restored to n (without spare).

The lead time follows an exponential distribution with rate β in both cases. At the time of replenishment, we discard the failed elements. The excess elements, after materialization, are kept as spares and used to replace defective elements. Thus, the maximal number of elements (including both working and spare) in the system is

$$B_{max} = n + H - k. \tag{1}$$

We define $X(t)$ as the number of elements (working and spare) at time t and let $Y(t)$ represent the restoration state at time t , which takes one of three values,

$$Y(t) = \begin{cases} 0, & \text{when repair and replenishment are OFF.} \\ 1, & \text{when repair is ON, replenishment is OFF,} \\ 2, & \text{when repair is ON, replenishment is ON.} \end{cases}$$

The CTMC $\{X(t), Y(t)\}_{t \geq 0}$ has finite state space

$$\Omega = \{(i, 0) \mid L + 1 \leq i \leq B_{max}\} \cup \{(i, 1) \mid L \leq i \leq H - 1\} \cup \{(i, 2) \mid 0 \leq i \leq H - 1\}.$$

The generator matrix Q is constructed using the following CTMC transition rates:

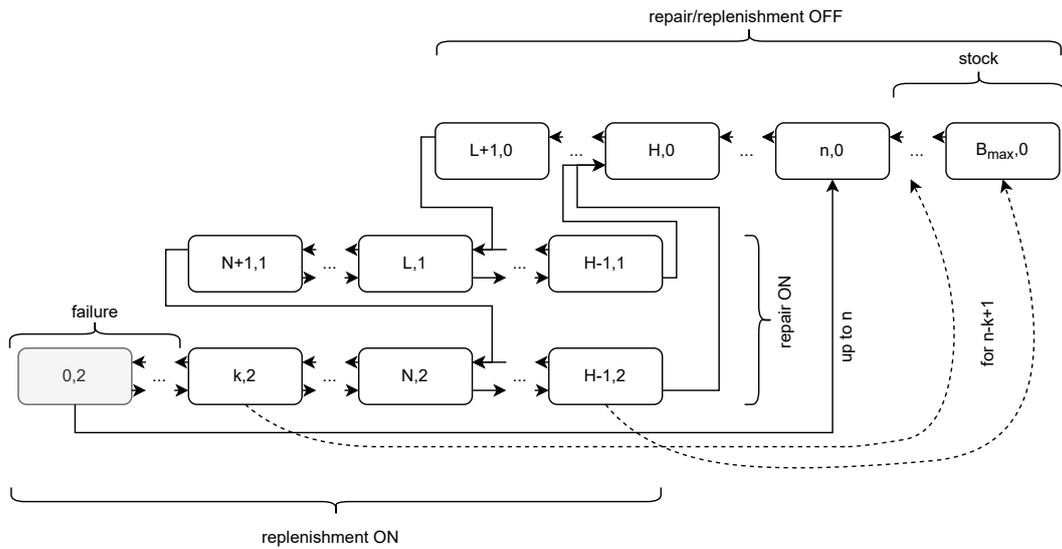


Figure 1: State transition diagram for the k -out-of- n : G model

(i) Degradation:

- in active state: $(i, 0) \rightarrow (i - 1, 0)$ with rate $\min(i, n)\lambda$, for $i = L + 2, \dots, B_{max}$;
- starting repair: $(L + 1, 0) \rightarrow (L, 1)$ with rate $(L + 1)\lambda$;
- in repair ON state: $(i, 1) \rightarrow (i - 1, 1)$ with rate $i\lambda$, for $i = N + 2, \dots, H - 1$;
- placing replenishment order: $(N + 1, 1) \rightarrow (N, 2)$ with rate $(N + 1)\lambda$;
- in repair ON and replenishment ON: $(i, 2) \rightarrow (i - 1, 2)$ with rate $i\lambda$, for $i = k, \dots, H - 1$;
- in failure state: $(i, 2) \rightarrow (i - 1, 2)$ with rate $i\theta$, for $i = 0, \dots, k - 1$.

(ii) Repair:

- if replenishment ON: $(i, 2) \rightarrow (i + 1, 2)$ with rate γ , for $i = 0, \dots, H - 2$;
- if only repair is ON: $(i, 1) \rightarrow (i + 1, 1)$ with rate γ , for $i = N + 1, \dots, H - 2$;
- when switching repair OFF: $(H - 1, j) \rightarrow (H, 0)$ with rate γ , for $j = 1, 2$.

(iii) Replenishment:

- in active state: $(i, 2) \rightarrow (i + n - k + 1, 0)$ with rate β , for $i = k, \dots, H - 1$;
- in failure state: $(i, 2) \rightarrow (n, 0)$ with rate β , for $i = 0, \dots, k - 1$.

All other transition rates (and corresponding off-diagonal elements of the generator matrix) are zero. The diagonal elements are negative to guarantee $\mathbf{Q}\mathbf{1} = \mathbf{0}$. The CTMC is irreducible and aperiodic and has unique steady-state stochastic vector $\boldsymbol{\pi} = \|\pi_{i,j}\|_{(i,j) \in \Omega}$ solving the linear system

$$\boldsymbol{\pi}\mathbf{Q} = \mathbf{0}, \quad \boldsymbol{\pi}\mathbf{1} = 1. \quad (2)$$

Using this steady-state distribution, we can illustrate how to derive some critical performance and reliability measures of the system in the Markov case.

1. Expected number of working elements when the repair is ON, E_{RON} .

$$E_{RON} = \sum_{i=N+1}^{H-1} i\pi_{i,1} + \sum_{i=0}^{H-1} i\pi_{i,2}. \quad (3)$$

2. Probability that the server is busy, P_B .

$$P_B = \sum_{i=N+1}^{H-1} \pi_{i,1} + \sum_{i=0}^{H-1} \pi_{i,2}. \quad (4)$$

We can find the probability that the server is idle as $P_I = 1 - P_B$.

3. Probability that the system is in the failed state, P_F .

$$P_F = \sum_{i=0}^{k-1} \pi_{i,2}. \quad (5)$$

4. Effective replenishment rate, E_{RR} .

$$E_{RR} = \beta \sum_{i=0}^{H-1} \pi_{i,2}. \quad (6)$$

Other performance/reliability metrics can be defined in a similar way.

Remark 1. Note that the steady-state distribution allows one to obtain the expected cycle length of various cycles that appear as the times at which the CTMC revisits some fixed state. In particular, the average cycle length $ET_{i,j}$, where $T_{i,j}$ is the (generic) time between the consecutive visits to the state $(i, j) \in \Omega$ of the CTMC is calculated as

$$ET_{i,j} = \left[-Q_{(i,j),(i,j)} \pi_{i,j} \right]^{-1}, \quad (7)$$

where the diagonal element $-Q_{(i,j),(i,j)}$ is the outgoing rate from state (i, j) and $\pi_{i,j}$ is the steady-state probability of that state.

3. SIMULATION MODEL

In order to study the sensitivity of the k -out-of- n : G model to the repair/replenishment distributions, we construct a simulation model using the DES approach within the framework of the so-called GSMPs [16]. This framework allows one to construct confidence intervals [27] for the desired performance estimates using regenerative simulation [28]. We note that historically, GSMP construction dates back to the 1970s and has several variations known as *service scheme* [29] or *reallocatable GSMP* [30].

To describe a GSMP, one needs to define a multi-dimensional stochastic process

$$\Theta = \{\mathbf{X}(t), \mathbf{T}(t)\}_{t \geq 0}, \quad (8)$$

which describes the system *state* vector $\mathbf{X}(t) \in \mathcal{X}$ evolving in time. The simulation time advances according to the *timers* (clocks) vector $\mathbf{T}(t) \geq \mathbf{0}$ among which the so-called *active timers* enumerated from the set $A(\mathbf{x}) \neq \emptyset$ (for any state $\mathbf{x} \in \mathcal{X}$) decrease linearly with rates given by a (state-dependent) vector $\mathbf{r}(\mathbf{x}) \geq \mathbf{0}$, positive for active timers:

$$r_i(\mathbf{x}) > 0 \text{ if and only if } i \in A(\mathbf{x}).$$

More formally,

$$\mathbf{T}(t+h) = \mathbf{T}(t) - h\mathbf{r}(\mathbf{X}(t)), \quad h \leq \min_{i \in A(\mathbf{X}(t))} T_i(t)/r_i(\mathbf{X}(t)).$$

At an *event* of type i , that is., a time epoch $t \geq 0$ such that $T_i(t-) = 0$, $i \in A(\mathbf{X}(t))$ (assuming events appearing singly), the state vector \mathbf{X} transitions from some state $\mathbf{x} \in \mathcal{X}$ into $\mathbf{x}' \in \mathcal{X}$ in accordance with transition probability matrix $P^{(i)} = \|P_{\mathbf{x},\mathbf{x}'}^{(i)}\|_{\mathbf{x},\mathbf{x}' \in \mathcal{X}}$, where

$$P_{\mathbf{x},\mathbf{x}'}^{(i)} = P\{\mathbf{X}(t) = \mathbf{x}' | \mathbf{X}(t-) = \mathbf{x}, T_i(t-) = 0\}. \quad (9)$$

After such a transition, the set of active timers may also change from $A(x)$ into $A(x')$. Thus, the *new active* timers $i' \in A(x') \setminus (A(x) \setminus \{i\})$ are initialized from some given density

$$f_{i'}(u, x, x', i) = P\{T_{i'}(t) \in du | X(t-) = x, X(t) = x', T_i(t-) = 0\}, \quad u > 0. \quad (10)$$

Since the state does not change between the events, tracking the system only at event epochs (standard for the DES) is necessary.

In this paper, we construct a GSMP with *simple* events [31], which means that the timer initialization density function depends only on the timer itself,

$$f_{i'}(u, x, x', i) \equiv f_{i'}(u).$$

This approach adds some complexity to the state space description, although it still keeps the model within the class of finite state space models. Additionally, note that the timer initialization densities are general, no longer restricting the model to exponential distributions.

The components of the state vector have the following meaning (according to the number of the component):

- $1, \dots, n$: an indicator of the state (working/broken) of the elements in active state (since there are at most n working elements);
- $n + 1, \dots, n + k - 1$: an indicator of the state (working/broken) of the elements in a failure state (since there can be at most $k - 1$ working elements);
- $n + k$: inventory state (number of spare elements);
- $n + k + 1$: indicator of the restoration state (0 – repair/replenishment OFF; 1 – repair ON, replenishment OFF; 2 – repair/replenishment ON).

Accordingly, the timers have the following functions:

- $1, \dots, n$: residual lifetimes of the elements in active state;
- $n + 1, \dots, n + k - 1$: residual lifetimes of the elements in failure state;
- $n + k$: residual repair time;
- $n + k + 1$: residual replenishment time.

That is, the system state vector X and timers vector T have the same dimension $n + k + 1$, where n and k are the model parameters (maximum number of working elements and failure threshold, respectively).

The timer initialization densities $f_{i'}(u)$ are then chosen according to the given p.d.f. for the corresponding event, that is, the life/repair/replenishment times. We assign them for the specific experiments separately.

A technical *restriction* of the model is that at the system state change (active/failure), the timers are initialized anew in a memoryless way. This assumption is due to the fact that the lifetime distribution is affected by the system state (elements' lifetimes have different distributions in active and failure states). Although it is possible to use a conditional distribution for the residual lifetime based on the already attained lifetime, we omit this possibility hereafter.

An important result for the finite state space GSMP models is the recurrence condition of the system states. As demonstrated in [31], if two or more clocks have clock setting distributions with an infinite mean, the recurrence of all states is not guaranteed. In contrast, all states are positive recurrent (that is., the hitting times of states have finite expectations regardless of the initial state) if the initialized clocks have finite means [31, Proposition 3.1]. To study this effect in our model, we perform a numerical experiment.

We define a GSMP model for the k -out-of- n : G system so that one-timer has a distribution with an infinite mean. To do so, we fix $n = 10$, $H = 8$, $L = 6$, $N = 4$, $k = 2$ and take all the timers

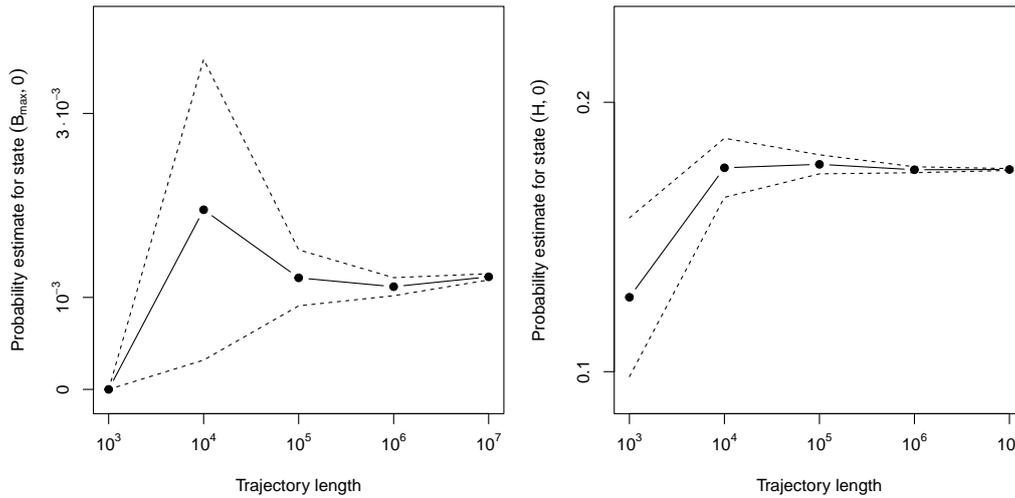


Figure 2: Regenerative estimates for the stationary probabilities of states $(B_{max}, 0)$ (left) and $(H, 0)$ (right) in a GSMP k -out-of- n : G model, at the confidence level 0.05, for given trajectory length 10^i for $i = 3, \dots, 7$. The repair times have heavy-tailed (Pareto) distribution (11) with $\alpha = 0.8$ and, consequently, infinite mean. The estimates obtained by *simulato* package.

to be exponentially distributed, apart from one being (type-II) Pareto distributed with shape parameter $\alpha = 0.8$, that is, having tail distribution function $\bar{F}(x) = 1 - F(x)$ of the following form,

$$\bar{F}(x) = x_0^\alpha (x + x_0)^{-\alpha}. \quad (11)$$

Namely, the system assigns exponentially distributed lifetimes to elements in the working and failure states, with rates $\lambda = 1$ and $\theta = 0.1$, respectively. Repair or replenishment times (for the corresponding cases) are also exponentially distributed with rates $\gamma = \beta = 10$. We consider two cases: one where the residual repair timer T_{n+k} has a heavy-tailed distribution with an infinite mean, and another where the residual replenishment timer T_{n+k+1} has such a distribution. Using the built-in capabilities of the *simulato* package[32] for R language, we look at the regenerative estimates of the stationary probabilities of the state $(H, 0)$ (in the original system) that corresponds to H working elements and no ongoing repair/replenishment, and the state $(B_{max}, 0)$. Note that the latter is only accessible through replenishment from the state $(H - 1, 2)$, and thus, one would expect this state to have infinite recurrence time if the replenishment timer has an infinite mean. We check this observation by plotting the estimate of such a probability with a growing number of transitions within the GSMP model.

Before doing so, we note that for validation purposes, we compared the accuracy of the GSMP model to the CTMC in case all timers have exponential distributions. The relative accuracy of the estimates did not exceed 0.4% for the GSMP trajectory length 10^7 . For the aforementioned model configuration, in the CTMC model, the states $(H, 0)$ and $(B_{max}, 0)$ had probabilities 0.1838 and 0.0013, respectively.

Now we analyze the GSMP model with the *replenishment* timer having heavy-tailed distribution and infinite mean. Figure 2 shows that the estimate for the state $(B_{max}, 0)$ is relatively small but positive. The state $(H, 0)$ has a positive steady-state probability. In both cases, the confidence interval decreases with increasing trajectory length. Note also that the values of the corresponding estimates are rather close to the purely exponential CTMC case mentioned above.

Figure 3 depicts the results for the system with a heavy-tailed *repair* timer distribution (having an infinite mean). As expected, the state $(H, 0)$ has a non-zero steady-state probability estimate with a decreasing confidence interval. The state $(B_{max}, 0)$ estimate is also positive but remains relatively small, falling below the simulation accuracy. This observation is somewhat counterintuitive. However, a closer look at the diagram of the possible model transitions given in Figure 1 can invoke a possible explanation for this phenomenon. In fact, the only possibility of

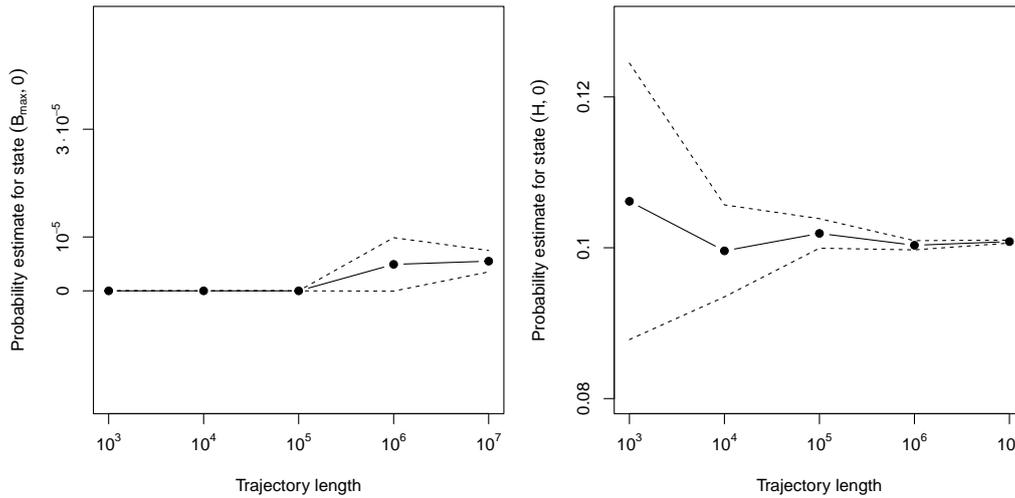


Figure 3: Regenerative estimates for the stationary probabilities of states $(B_{max}, 0)$ (left) and $(H, 0)$ (right) in a GSMP k -out-of- $n : G$ model, at the confidence level 0.05, for given trajectory length 10^i for $i = 3, \dots, 7$. The repair times have heavy-tailed (Pareto) distribution (11) with $\alpha = 0.8$ and, consequently, infinite mean.

entering the state $(B_{max}, 0)$ is through the state $(H - 1, 2)$. The latter is only reachable from states with fewer working elements due to repair. Thus, if the repair time has infinite mean, the state $(B_{max}, 0)$ may be suspected to be null recurrent.

Now we consider a system in which *both* the repair and replenishment timers are Pareto distributed. In this case, the model can enter the state $(B_{max}, 0)$ only through at least two sequential events: repair to enter $(H - 1, 2)$ and replenish to enter $(B_{max}, 0)$. If both timers have an infinite mean, it is more likely that the degradation event interrupts this chain of two heavy-tailed events, and following the arguments in [31], one may suspect the state $(B_{max}, 0)$ to be transient. To observe this phenomenon, we plot the stationary probability estimate of the state $(B_{max}, 0)$ for the cases when both timers have Pareto distribution (11) with $\alpha = 0.8$ vs. $\alpha = 1.2$. While in the latter case, this state has a small positive probability (having an order of the simulation accuracy), the probability estimate is negligible in the former case. We extend this investigation by plotting the probability estimates of the number of working elements for two trajectory lengths, 10^4 and 10^7 . Figure 5 shows that most states appear transient in the model with $\alpha = 0.8$, except for the zero states where no elements are working. We also see that the estimate of the probability of the zero state increases with increasing trajectory length, that is., 0.6795 for 10^4 and 0.9505 for 10^7 transitions.

CONCLUSION

In this paper, we consider the so-called k -out-of- $n : G$ model with a single repair element, stock of spare elements, state-dependent replenishment policy, and (threshold-based) state-dependent element degradation. While the Markov case does not display any difficulties, since the model is a finite state space Markov chain, in the general case positive recurrence of the states is questionable. We investigated the general model by simulation and demonstrated the effects of heavy tails (for repair and replenishment time distributions) on the probabilities of the system states. As we observed from the results of numerical experiments, the model seems to possess null-recurrence and transience for some states, which is similar to the effects reported in [31]. A rigorous proof of those effects is a promising direction for further research.

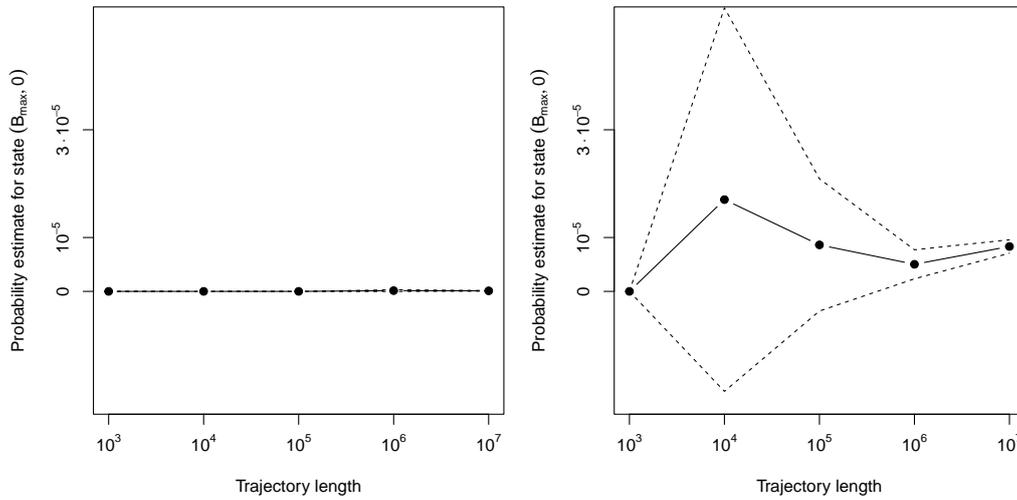


Figure 4: Regenerative estimates for the stationary probabilities of states $(B_{max}, 0)$ in a GSMP k -out-of- $n : G$ model, at the confidence level 0.05, for given trajectory length 10^i for $i = 3, \dots, 7$. Both the repair and replenishment times have heavy-tailed (Pareto) distribution (11) with infinite mean for $\alpha = 0.8$ (left) and finite mean for $\alpha = 1.2$ (right).

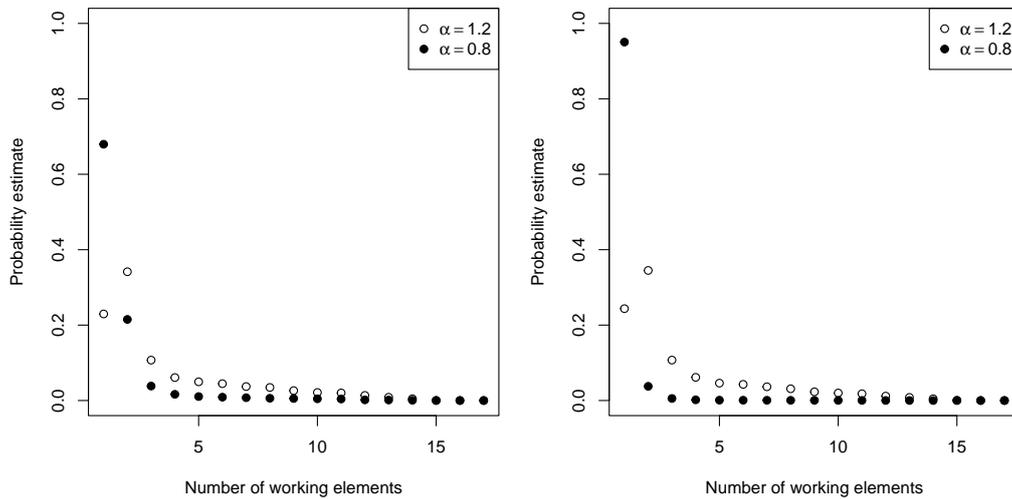


Figure 5: Relative frequencies vs. number of working elements in the GSMP k -out-of- $n : G$ model for trajectory length 10^7 . Both the repair and replenishment times have heavy-tailed (Pareto) distribution (11) with infinite mean for $\alpha = 0.8$ (black dots) and finite mean for $\alpha = 1.2$ (white dots) for simulation trajectory of length 10^4 (left) and 10^7 (right).

ACKNOWLEDGMENTS

This work was partially supported by the Moscow Center for Fundamental and Applied Mathematics (recipient A. Rummyantsev).

REFERENCES

- [1] Elvis Rojas et al. "Towards a Model to Estimate the Reliability of Large-Scale Hybrid Supercomputers". In: *Euro-Par 2020: Parallel Processing*. Springer International Publishing, 2020, pp. 37–51. ISBN: 9783030576752. DOI: 10.1007/978-3-030-57675-2_3.
- [2] Heping Jia et al. "Reliability evaluation of power systems with multi-state warm standby and multi-state performance sharing mechanism". In: *Reliability Engineering & System Safety* 204 (Dec. 2020), p. 107139. ISSN: 0951-8320. DOI: 10.1016/j.ress.2020.107139.
- [3] *Cochin International Airport Ltd. CIAL's green energy generation touches 25 Cr. Units*. URL: <https://www.cial.aero/news-Updates/CIAL-s-green-energy>.
- [4] Sreenath Sukumaran and K. Sudhakar. "Fully solar powered airport: A case study of Cochin International airport". In: *Journal of Air Transport Management* 62 (July 2017), pp. 176–188. ISSN: 0969-6997. DOI: 10.1016/j.jairtraman.2017.04.004.
- [5] A. Krishnamoorthy, P. Ushakumari, and B. Lakshmy. "k-out-of-n-system with repair: The N-policy". In: *Asia-Pacific Journal of Operational Research* 19 (May 2002), pp. 47–61.
- [6] Achyutha Krishnamoorthy. "A Reliability-Inventory Problem Under N-policy of replenishment of component". In: (2021). DOI: 10.24412/1932-2321-2021-465-73-87.
- [7] A. Krishnamoorthy and P. V. Ushakumari. "Reliability of ak-out-of-n system with repair and retrieval of failed units". In: *Top* 7.2 (Dec. 1999), pp. 293–304. ISSN: 1863-8279. DOI: 10.1007/bf02564728.
- [8] Srinivas R. Chakravarthy, A. Krishnamoorthy, and P. V. Ushakumari. "A k-out-of-n reliability system with an unreliable server and phase type repairs and services: the (N, T) policy". In: *International Journal of Stochastic Analysis* 14.4 (Jan. 2001), pp. 361–380. ISSN: 2090-3340. DOI: 10.1155/s1048953301000326.
- [9] P V Ushakumari. "k-out-of-n System with Repair under Single/Bulk Service". en. In: *Degres Journal* 9.7 (2024), pp. 27–36.
- [10] Shan Gao, Jinting Wang, and Qin Chen. "Reliability Evaluation for a Circular Con/k/n:F System With a Novel Differential Repair Policy". In: *IEEE Transactions on Reliability* (2025), pp. 1–14. ISSN: 1558-1721. DOI: 10.1109/tr.2024.3524329.
- [11] A. Krishnamoorthy, Vishwanath C. Narayanan, and T. G. Deepak. "Optimal utilization of service facility for ak-out-of-n system with repair by extending service to external customers in a retrieval queue". In: *Journal of Applied Mathematics and Computing* 25.1–2 (Sept. 2007), pp. 389–405. ISSN: 1865-2085. DOI: 10.1007/bf02832364.
- [12] Vladimir Rykov et al. "On reliability function of a k-out-of-n system with general repair time distribution". In: *Probability in the Engineering and Informational Sciences* 35.4 (May 2020), pp. 885–902. ISSN: 1469-8951. DOI: 10.1017/s0269964820000285.
- [13] Ameneh Farahani, Ahmad Shoja, and Hamid Tohidi. "Markov and semi-Markov models in system reliability". In: *Engineering Reliability and Risk Assessment*. Elsevier, 2023, pp. 91–130. ISBN: 9780323919432. DOI: 10.1016/b978-0-323-91943-2.00010-1.
- [14] Oliver C. Ibe. *Markov processes for stochastic modeling*. eng. 2nd ed. Elsevier insights. London: Elsevier, 2013. ISBN: 978-0-12-407839-0.
- [15] Peter J Haas. *Stochastic Petri nets modelling, stability, simulation*. English. New York: Springer, 2002. ISBN: 978-0-387-21552-5. (Visited on 08/17/2014).

- [16] P. W. Glynn. "A GSMP formalism for discrete event systems". In: *Proceedings of the IEEE* 77.1 (1989), pp. 14–23. doi: 10.1109/5.21067.
- [17] Ji-Eun Byun, Hee-Min Noh, and Junho Song. "Reliability growth analysis of k-out-of-N systems using matrix-based system reliability method". In: *Reliability Engineering & System Safety* 165 (Sept. 2017), pp. 410–421. issn: 0951-8320. doi: 10.1016/j.res.2017.05.001.
- [18] Nika Ivanova. "Modeling and Simulation of Reliability Function of a k-out-of-n:F System". In: *Distributed Computer and Communication Networks: Control, Computation, Communications*. Springer International Publishing, 2020, pp. 271–285. isbn: 9783030662424. doi: 10.1007/978-3-030-66242-4_22.
- [19] N. M. Ivanova. "On Steady State Reliability and Sensitivity Analysis of a k-out-of-n System Under Full Repair Scenario". In: *Distributed Computer and Communication Networks: Control, Computation, Communications*. Springer Nature Switzerland, 2022, pp. 422–434. isbn: 9783031232077. doi: 10.1007/978-3-031-23207-7_33.
- [20] Xiaohu Li, Ming J. Zuo, and Richard C.M. Yam. "Reliability analysis of a repairable k-out-of-n system with some components being suspended when the system is down". In: *Reliability Engineering & System Safety* 91.3 (Mar. 2006), pp. 305–310. issn: 0951-8320. doi: 10.1016/j.res.2005.01.010.
- [21] Yuanyuan Zhang, Wenqing Wu, and Yinghui Tang. "Analysis of an k-out-of-n:G system with repairman's single vacation and shut off rule". In: *Operations Research Perspectives* 4 (2017), pp. 29–38. issn: 2214-7160. doi: 10.1016/j.orp.2017.02.002.
- [22] Hui Wu, Yan-Fu Li, and Christophe Bérenguer. "Optimal inspection and maintenance for a repairable k-out-of-n: G warm standby system". In: *Reliability Engineering & System Safety* 193 (Jan. 2020), p. 106669. issn: 0951-8320. doi: 10.1016/j.res.2019.106669.
- [23] J. Banks. *Discrete-event System Simulation*. Prentice Hall, 2010. isbn: 9780136062127. url: <https://books.google.ru/books?id=cqSNnmrqqbQC>.
- [24] M. Averill Law and W.D. Kelton. *Simulation Modeling and Analysis*. 4th ed. New York: McGraw Hill, 2007.
- [25] A. N. Dudin, A. Krishnamoorthy, and V. C. Narayanan. "Idle time utilization through service to customers in a retrial queue maintaining high system reliability*". In: *Journal of Mathematical Sciences* 191.4 (May 2013), pp. 506–517. issn: 1573-8795. doi: 10.1007/s10958-013-1336-3.
- [26] A. Krishnamoorthy, V. C. Narayanan, and T. G. Deepak. "Reliability of a k-out-of-n system with repair by a service station attending a queue with postponed work". In: *International Journal of Reliability, Quality and Safety Engineering* 14.04 (Aug. 2007), pp. 379–398. issn: 1793-6446. doi: 10.1142/s0218539307002702.
- [27] Shane G. Henderson and Peter W. Glynn. "Regenerative steady-state simulation of discrete-event systems". en. In: *ACM Transactions on Modeling and Computer Simulation* 11.4 (Oct. 2001), pp. 313–345. issn: 10493301. doi: 10.1145/508366.508367. (Visited on 04/19/2018).
- [28] Søren Asmussen and Peter W. Glynn. *Stochastic simulation: algorithms and analysis*. en. Stochastic modelling and applied probability 57. OCLC: ocn123113652. New York: Springer, 2007. isbn: 978-0-387-69033-9.
- [29] D. König, V. V. Rykov, and V. Schmidt. "Stationary queuing systems with dependencies". en. In: *Journal of Soviet Mathematics* 21.6 (1983), pp. 938–994. issn: 0090-4104, 1573-8795. doi: 10.1007/BF01089194. (Visited on 11/15/2018).
- [30] Masakiyo Miyazawa. "Insensitivity and Product-Form Decomposability of Reallocatable GSMP". In: *Advances in Applied Probability* 25.2 (1993). Publisher: Applied Probability Trust, pp. 415–437. issn: 00018678. doi: 10/d5fqss. (Visited on 03/26/2020).
- [31] Peter W. Glynn and Peter J. Haas. "On Transience and Recurrence in Irreducible Finite-State Stochastic Systems". en. In: *ACM Transactions on Modeling and Computer Simulation* 25.4 (May 2015), pp. 1–19. issn: 10493301. doi: 10/f767qh. (Visited on 03/26/2020).

- [32] A. Rummyantsev and S. Astafiev. *simulato: Simulation framework for GSM processes*. URL: <https://R-Forge.R-project.org/projects/simulato/>.